Physica C 509 (2015) 22–28

Contents lists available at ScienceDirect

Physica C

journal homepage: www.elsevier.com/locate/physc

Scattering-matrix approach to the theory of Josephson transport in mesoscopic multiterminal nodes

D.A. Savinov *

Institute for Physics of Microstructures RAS, 603950, Russia
Lobachevsky State University of Nizhni Novgorod, Nizhni Novgorod 603950, Russia

ARTICLE INFO

Article history:
Received 27 October 2014
Received in revised form 26 November 2014
Accepted 3 December 2014
Available online 13 December 2014

Keywords:
Multiterminal Josephson nodes
Transmission-matrix formalism
Scattering problem
Subgap energy spectrum
Current-phase relation

ABSTRACT

Within the Bogoliubov–de Gennes theory we have studied the transport properties of disordered Josephson systems with several superconducting electrodes weakly coupled through a normal region. The scattering-matrix approach has been applied for arbitrary geometries of the normal region to analyze the quasi-particle spectra and current-phase relations (CPRs). The dependence of CPRs on the system geometry and its lengths reveals itself in the coefficients of the scattering matrix in the connecting normal area. This approach is applicable to mesoscopic superconducting structures with a normal weak link which appears to be small compared to the superconducting coherence length. The possible applications of our results and typical experimental examples have been discussed.

1. Introduction

The study of the effects of weak superconductivity has been a challenging problem for several decades. A theory of weakly coupled superconductors was formulated by Brian D. Josephson in 1962 [1]. Josephson predicted that the system of the two superconductors separated by a thin ($\sim$10 A) dielectric barrier (tunnel junction) would sustain a direct supercurrent through the junction. This Josephson phenomena can occur not only in tunnel junctions but also in any superconductor with a weak link, e.g., in a constriction-type junction [2,3], in a superconductor–normal metal–superconductor (S–N–S) junction [4–6], or in a superconductor–two-dimensional electron gas–superconductor (S–2-DEG–S) junction [7]. Valuable applications of weak-link effects, such as the formation of π-junctions, originate in superconductor–ferromagnet–superconductor (S–F–S) junction [8]. The experimental evidence for the existence of such exotic state in Josephson junctions was presented, e.g., in [9,10]. Though, within our paper we do not consider ferromagnetic weak links in Josephson junctions and focus on the manifestation of the Josephson effects in structures of several (three or more) weakly coupled superconducting banks [11–25]. There are many fascinating results have been derived in this field, e.g., nontrivial current-phase relations (CPRs), effects of nonlocal weak coupling, bistable states, magnetic flux transfer, voltage-induced Shapiro-like steps, etc. (see [11–18] for details). The superconducting multiterminal structures can be used to fabricate the specific interference devices, e.g., Y-shaped Andreev interferometers [19,21–24]. Experiments with such structures were recently performed by Meschke et al. in their work [21] and the theoretical calculations were carried out in [19,22–24]. The possibility of a φ-junction was recently demonstrated in a three-terminal Josephson node [25]. The obvious interest to these multiterminal systems is stimulated by the possibility to reduce the parasitic inductance of the leads which results in various inertia effects and negative influence of external electromagnetic noise. Also these systems can be particularly attributed by its possible application to the Josephson quantum bits (qubits). In particular, qubits with conventional Josephson junctions were considered in [26–29]. Besides, the possibility for building a superconducting qubit on the basis on multiterminal Josephson structures was recently discussed in [25].

In the vicinity of the critical temperature Θ_c the theoretical study of multiterminal Josephson nodes can be carried out within the Ginzburg–Landau approach as it was presented in [11,12,14,15,18,25]. For rather low temperatures Θ (when $\Theta_c - \Theta \ll \Theta_c$) one should develop a microscopic theory. In particular, the tunneling Hamiltonian formalism [30] can be used to describe the transport properties of multiterminal Josephson systems. Such microscopic analysis was, e.g., carried out for three-terminal structure in [25]. The Eilenberger equation [31]

* Address: Institute for Physics of Microstructures RAS, 603950, Russia. Tel.: +7 831 417 94 85.
E-mail address: savinovda@ipm.sci-nnov.ru

http://dx.doi.org/10.1016/j.physc.2014.12.002
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and the Usadel equation [32] are also employed in this low-temperature case for clean limit $\ell_S \gg \xi$ [13,16,17] and dirty regime $\ell_S \ll \xi$ [20,22–24], respectively, where ℓ_S is the mean free path in superconductor and ξ is the superconducting coherence length. In the framework of these models the description of transport properties in multiterminal Josephson systems takes into account the peculiarities of the system geometry. Therefore, the results strongly depend on the system lengths and its configuration. Below we briefly discuss two different examples which confirm this statement. (i) Within the framework of Eilenberger equation Zareyan and Omelyanchouk studied the CPRs in a Josephson system with four weakly coupled bulk superconducting banks [13]. As a result, they showed a strong dependence of CPRs on the length and width of 2-DEG rectangular which represents a ballistic junction for the superconducting banks. (ii) On the basis of Usadel equation the voltage-induced Shapiro-like steps were predicted in a three-terminal Josephson structure [20]. The authors of [20] studied a mesoscopic multiterminal system formed by a straight diffusive S–N–S junction and a superconducting tunneling probe attached to the normal wire. Similar to the previous example, the analysis of Cuevas and Pothier was considered only for a certain geometry of the system and could not be generalized for another configuration of the normal region. Moreover, these theoretical approaches are usually rather complicated by tedious mathematical manipulations.

Here we develop an alternative formalism for theoretical analysis of transport properties in a Josephson system with several superconducting electrodes weakly coupled by a normal area. This formalism is based on using the scattering matrix inside the junction. It allows us to study the special technique which can be applied for different types of system geometry and should not take account of the peculiarities of quasi-particle motion in the normal region. For this aim we are not interested in the statistical properties of the scattering matrix, i.e. we just solve the Bogoliubov–de Gennes equations and do not average the treatment over an ensemble of impurity configurations. We claim that such a theoretical approach demands essentially smaller mathematical manipulations than the models based on the solution of the Eilenberger or Usadel equations.

Our article is mainly devoted to the analysis of a multiterminal setup which consists of a disordered normal region embedded between three superconducting arms (Josephson triode). The typical hybrid system is schematically shown in Fig. 1. In the framework of Bogoliubov–de Gennes theory we study the transport properties of such hybrid mesoscopic system in a short-disordered-junction regime, i.e. the typical length of the normal region L does not exceed the superconducting coherence length ξ and appears to be larger than the elastic mean free path $\ell : \ell \ll L \ll \xi$. This theory is directly applicable to superconductors in the clean limit $\ell_S \gg \xi$. Therefore, we assume that the only scattering in the superconducting banks consists of Andreev reflection at the *SN* interfaces. We also study the case when the disorder is contained entirely in the vicinity of the normal-region center which couples the ideal (impurity-free) normal leads N_1, N_2, and N_3. Our goal is to calculate the quasi-particle spectra and then the CPRs within the transmission-matrix formalism introduced earlier by Beenakker for the S–N–S junction [33]. We would like to emphasize that the qualitative results of our analysis are not dependent on whether the disorder extends into the superconducting arms or not, as well as the normal reflections at the *SN* interfaces exist or not. Indeed, such non-idealities can be controlled by the coefficients of the scattering matrix in the normal area (see also the discussion in [33]).

Starting from this formalism, we find the subgap quasiparticle spectrum and the CPR for a particular case of a single-mode regime, which appears to be justified when the transverse lengths

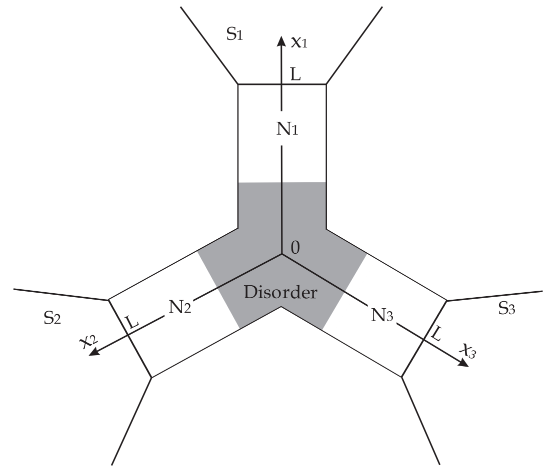

Fig. 1. Schematic view of a hybrid system, consisting of three superconducting banks (S_1, S_2 and S_3) weakly coupled by a normal region. The normal region contains disordered area (dark) and impurity-free normal leads N_1, N_2 and N_3.

of each normal lead (N_1, N_2, and N_3) become of the order of λ_F, where λ_F is the Fermi wave length. Till recently this particular case has appeared to be just a model problem. It is interesting only from the theoretical point of view and hardly achieved in the experiment because the typical values of λ_F appear to be very small: they are of the order of 1 *A* in normal metals and of the order of 10 nm in 2-DEG of typical semiconducting heterostructures. However, very recently a single-mode multiterminal structure (with four branches), known as a beam splitter interferometer, was considered in [34]. In this work Beenakker proposed an experiment for Bogoliubov quasi-particles in 2-DEG that is the condensed matter analogue of the way in which Majorana fermions are searched for in particle physics: by detecting their pairwise annihilation upon collision. Besides, such beam splitter interferometers have been earlier implemented using the quantum Hall edge channels of a 2-DEG as chiral (unidirectional) wave guides, to realize the electronic analogues of the Hanbury Brown–Twiss (HBT) experiment [35–37] and the Hong–Ou–Mandel (HOM) experiment [38,39]. The system we propose here (as well as Beenakker considered in [34]) differs in one essential aspect: before reaching the beam splitter, the quasi-particles are Andreev reflected at the superconducting banks. Andreev reflection in the single-mode regime has been earlier reported in the superconducting contacts to a 2-DEG located in InAs/Al_xGa_{1-x}Sb and Ga_xIn_{1-x} As/InP heterostructures (see [40] and [41], respectively), as well as in graphene–S and S–graphene–S contacts (see [42] and [43,44], respectively). Thus, we state that the single-mode consideration for our theoretical approach appears to be rather realistic when the normal region represents 2-DEG located in the high-mobility heterostructures mentioned slightly above or graphene.

As a next step, we study the multiple-mode case which appears to be valuable when the transverse lengths of the normal leads are assumed to be much larger than λ_F. Such multiple-mode Josephson nodes with several superconducting arms were partially considered in [13,16,17,20,22–24] discussed above. In this manuscript we develop a perturbation theory for the analysis of the Josephson currents in the superconducting arms and find out an approximate expression for CPR. Besides, the supercurrents are derived for rather low temperatures when the second and combinative harmonic terms in the CPR become extremely important.

In the framework of two cases (single-mode and multiple-mode) we get the CPRs of the Josephson triode which do not depend on the junction lengths and its geometry. The superconducting currents are completely determined by the coefficients of the scattering matrix in the normal area. There are two main