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Shielding current analysis by current-vector-potential method: Application to HTS film with multiply-connected structure

A. Kamitani [a,*], T. Takayama [a], S. Ikuno [b]

[a] Yamagata University, 4-3-16, Johnan, Yonezawa, Yamagata 992-8510, Japan
[b] Tokyo University of Technology, 1404-1, Katakura, Hachioji, Tokyo 192-0982, Japan

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ABSTRACT

The performance of the virtual voltage method is compared with that of the conventional method in which integral forms of Faraday's law along crack surfaces are treated as natural boundary conditions. As a result, it is found that there is a significant difference between numerical solutions by the two methods. In this sense, not the conventional method but the virtual voltage method should be employed to the shielding current analysis in a high-temperature superconducting film with cracks. By means of the virtual voltage method, the influence of a crack on the inductive method is investigated numerically. The results of computations show that, if the threshold current changes remarkably from its ambient value, a part of a crack is contained in the projection of the field-generating coil onto the film surface. Furthermore, the applicability of the inductive method to the crack detection is investigated numerically.

1. Introduction

The inductive method [1,2] has been widely used for measuring the critical current density j_C of high-temperature superconducting (HTS) films. In the method, while applying an ac current $I(t) = I_0 \sin 2\pi ft$ in a coil placed just above an HTS film, the third-harmonic voltage $V_3 \sin(6\pi ft + \theta_3)$ induced in the coil is monitored. According to the experimental results, V_3 abruptly develops when I_0 exceeds a threshold current I_T [1,2].

By assuming the critical state model, Mawatari et al. [2] performed a theoretical analysis to get the following equation: $j_C = 2F(r_m)I_T/b$. Here, $F(r_m)$ denotes the maximum of the primary coil-factor function and b is the thickness of an HTS film. By substituting the measured value of I_T into this equation, j_C can be estimated. This is the basic idea of the inductive method.

On the other hand, it is not clear whether or not the inductive method is applicable to an HTS film containing cracks. In order to calculate the shielding current density in an HTS film with cracks, the authors proposed the virtual voltage method [3,4]. Although the virtual voltage method was successfully applied to the numerical simulation of the permanent magnet method [5], its performance has not yet been investigated in detail.

The purpose of the present study is to assess the performance of the virtual voltage method and to numerically investigate the influence of a crack on the inductive method.

2. Shielding current analysis

2.1. Initial-boundary-value problem

A schematic view of the inductive method is shown in Fig. 1. In the inductive method, an M-turn coil is placed above an HTS film of thickness b so that the symmetry axis of the coil may be vertical to the film surface. By taking the thickness direction as z-axis and choosing the centroid of the film as the origin, let us use the Cartesian coordinate system $\langle O: \boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z \rangle$. In terms of the coordinate system, the symmetry axis of the coil can be written as $(x, y) = (x_A, y_A)$. In the following, $\boldsymbol{n}$ and $\boldsymbol{t}$ denote a normal unit vector and a tangent unit vector on an arbitrary curve in the xy plane, respectively. Also, $\boldsymbol{x}$ and $\boldsymbol{x}'$ are position vectors of two points in the xy plane.

We first assume that an HTS film has a square cross section Ω of side length a and that it contains a crack whose cross section is a line segment connecting two points, $(x_c, y_c \pm L_c/2)$, in the xy plane. For this case, the boundary $\partial\Omega$ of Ω is composed of the outer boundary C_0 and the inner boundary C_1 (see Fig. 1). Apparently, C_1 is the crack surface. We further assume that the coil has a rectangular cross section of inner radius R_{in}, outer radius R and height H. Also, the distance between the coil bottom and the film surface is denoted by L.

As is well known, the shielding current density $\boldsymbol{j}$ and the electric field $\boldsymbol{E}$ are closely related to each other in HTS films. As the relation, the following power law [6–10] is assumed:

$$\boldsymbol{E} = E(|\boldsymbol{j}|)[\boldsymbol{j}/|\boldsymbol{j}|], \quad E(j) = E_C[j/j_C]^N.$$

* Corresponding author. Tel.: +81 238 26 3331; fax: +81 238 26 3789.
E-mail address: kamitani@yz.yamagata-u.ac.jp (A. Kamitani).

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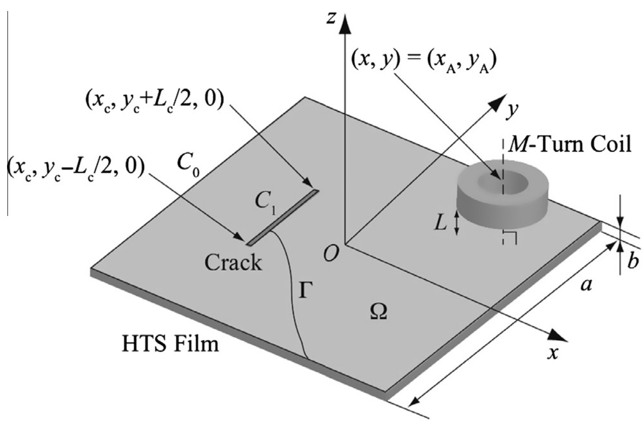

Fig. 1. A schematic view of the inductive method.

Here, E_C and j_C denote the critical electric field and the critical current density, respectively, and N is a positive constant.

Under the thin-plate approximation, there exists a scalar function $S(\boldsymbol{x}, t)$ such that $\boldsymbol{j} = (2/b)[\nabla \times (S\boldsymbol{e}_z)]$ and its time evolution is governed by the following integrodifferential equation [3,4,6,11]:

$$\mu_0 \partial_t(\widehat{W}S) + (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z = -\partial_t \langle \boldsymbol{B} \cdot \boldsymbol{e}_z \rangle, \tag{1}$$

where $\langle \ \rangle$ is an average operator over the thickness and $\widehat{W}$ is an operator defined by

$$\widehat{W}S \equiv \int\int_\Omega Q(|\boldsymbol{x} - \boldsymbol{x}'|)S(\boldsymbol{x}', t)\ d^2\boldsymbol{x}' + \frac{2S(\boldsymbol{x}, t)}{b}.$$

Here, the function $Q(r)$ is given by

$$Q(r) = -(\pi b^2)^{-1}[r^{-1} - (r^2 + b^2)^{-1/2}].$$

The initial and boundary conditions to (1) are assumed as follows [3,4]:

$$S = 0 \text{ at } t = 0,$$
$$S = 0 \text{ on } C_0,$$
$$\frac{\partial S}{\partial s} = 0 \text{ on } C_1,$$
$$\gamma[\boldsymbol{E}] \equiv \oint_{C_1} \boldsymbol{E} \cdot \boldsymbol{t}\ ds = 0,$$

where s is an arclength. Note that $\gamma[\boldsymbol{E}] = 0$ is the integral form of Faraday's law on the crack surface C_1.

By solving (1) together with the initial and boundary conditions, we can determine the time evolution of the shielding current density $\boldsymbol{j}$. Throughout the present study, the parameters are assumed as follows: $a = 20$ mm, $b = 600$ nm, $j_C = 1$ MA/cm^2, $E_C = 1$ mV/m, $N = 20$, $(x_c, y_c) = (0\text{ mm}, 0\text{ mm})$, $f = 1$ kHz, $M = 400$, $R_{in} = 1.5$ mm, $R = 2.5$ mm, $H = 2$ mm, and $L = 0.5$ mm.

2.2. Virtual voltage method

Throughout this section, the superscript (n) denotes a value at time $t = n\Delta t$, where Δt is a time step size. If the initial-boundary-value problem of (1) is discretized with the backward Euler method, $S^{(n)}$ becomes a solution S of the following nonlinear boundary-value problem:

$$G(S) \equiv \mu_0 \widehat{W}S + \Delta t\ (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z - u = 0 \text{ in } \Omega, \tag{2}$$

$$S \in H(\overline{\Omega}), \tag{3}$$

$$\gamma[\boldsymbol{E}] = 0, \tag{4}$$

where $u = \mu_0 \widehat{W}S^{(n-1)} - \langle \boldsymbol{B}^{(n)} - \boldsymbol{B}^{(n-1)} \rangle \cdot \boldsymbol{e}_z$ and $\overline{\Omega} = \Omega \cup \partial\Omega$. Also, the function space $H(\overline{\Omega})$ is defined by $H(\overline{\Omega}) \equiv \{w(\boldsymbol{x}) : w = 0$ on $C_0,\ \partial w/\partial s = 0$ on $C_1\}$.

After a straightforward calculation, we get the weak form that is equivalent to (2) and (4). Note that (4) is incorporated into the weak form. In other words, (4) is treated as a natural boundary condition. As is well known, a natural boundary condition is not exactly satisfied by a numerical solution of a weak form. Hence, even if the weak form is numerically solved with the essential boundary condition (3), the resulting numerical solution does not accurately satisfy (4). In order to investigate the residual in $\gamma[\boldsymbol{E}]$, the numerically evaluated value $N[\boldsymbol{E}]$ of $\gamma[\boldsymbol{E}]$ is determined by using the numerical integration. The results of computations show that $N[\boldsymbol{E}]$ does not vanish but take a relatively large value [3]. Hereafter, the numerical method for solving the weak form with (3) is called the conventional method.

For the purpose of having $N[\boldsymbol{E}] = 0$ exactly fulfilled, the authors proposed the virtual voltage method [3,4]. The basic idea of this method is explained as follows: a virtual voltage ϕ_V is applied along the crack surface C_1 so as to have $N[\boldsymbol{E}] = 0$ satisfied. The detailed explanation of the virtual voltage method is given in Appendix A.

Let us compare the performance of the virtual voltage method with that of the conventional method. To this end, we numerically evaluate the toroidal current I_T defined by $I_T \equiv b\int_\Gamma \boldsymbol{j} \cdot \boldsymbol{n}\ ds$. Here, the integration is carried out along an arbitrary curve Γ connecting the outer boundary C_0 with the inner boundary C_1 (see Fig. 1). The time dependence of I_T is determined by means of the two methods and the results of computations are depicted in Fig. 2. For the case with $0.1 < \text{mod}(ft, 1) < 0.3$ or $0.6 < \text{mod}(ft, 1) < 0.8$, there is a significant difference between the toroidal currents determined by the two methods. This result suggests that an accurate solution cannot be obtained by means of the conventional method. In fact, $\boldsymbol{j}$-distributions by the two methods show slightly different patterns especially near both ends of the crack (see Figs. 3(a) and 3(b)).

3. Numerical simulation of inductive method

On the basis of the virtual voltage method, a numerical code has been developed for analyzing the time evolution of the shielding current density. By executing the code, the threshold current I_T can be easily evaluated. In this section, we numerically investigate the following two problems:

- How is the inductive method affected by a crack?
- Is the inductive method applicable to the crack detection?

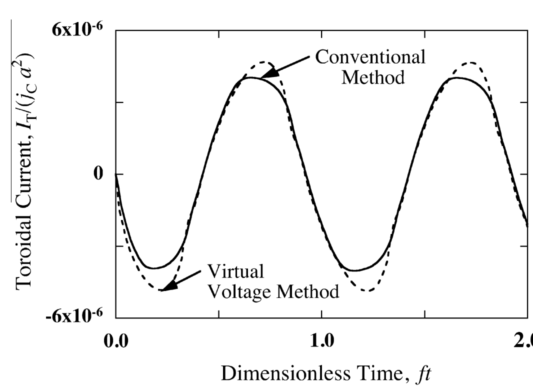

Fig. 2. The time dependence of the toroidal current I_T for the case with $I_0 = 400$ mA, $L_c = 16$ mm and $(x_A, y_A) = (0\text{ mm}, 0\text{ mm})$.