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Experimental tests of the modified Ginzburg–Landau theory for superconductors with reduced phase stiffness

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ABSTRACT

Recently, a modified version of the Ginzburg–Landau theory which takes into account the reduced phase stiffness of the underdoped cuprates has been developed. In this work we propose new experimental tests of this theory making use of neutron scattering and muon spin rotation experiments.

1. Introduction

One of the central questions in the physics of cuprate superconductors deals with the origin of the pseudogap "phase". According to one class of theories, the pseudogap is a precursor to the low-temperature superconducting state. Within this picture, in the pseudogap region the amplitude of the order parameter has a finite value, but the phase stiffness vanishes. When cooling into the superconducting state from the pseudogap, we should therefore expect that the phase stiffness, though finite, is much smaller than the amplitude stiffness.

In Refs. [1,2] we have developed a modified version of the Ginzburg–Landau (GL) phenomenological theory of the superconducting state of underdoped cuprates which differs from the conventional theory by allowing for different phase and amplitude stiffness. For this purpose, we have introduced two different coherence lengths, the amplitude coherence length ξ and the phase coherence length $\xi_\perp$. We have postulated that the ratio of coherence lengths $s = \xi_\perp/\xi$ is unity deep inside the superconducting region and vanishes at the boundary between the superconductor and the pseudogap "phase", see Fig. 1.

In particular, in Refs. [1,2] we have studied the following modified superconducting free-energy density

$$\frac{\delta\mathcal{F}_s}{\mu_0 H_c^2} = -f^2 + \frac{1}{2}f^4 + \xi^2(\nabla f)^2 + s^2\xi^2 f^2\left(\nabla\theta + \frac{2\pi}{\Phi_0}\mathbf{A}\right)^2,$$

where we have represented the macroscopic wave function in terms of its phase and amplitude as $\Psi(\mathbf{r}) = \Psi_\infty f(\mathbf{r})\, e^{i\theta(\mathbf{r})}$. Within the modified GL theory we have calculated the approximate magnetic field profile in the vortex lattice and several thermodynamic properties, such as the lower and upper critical fields H_{c1}, H_{c2}, and the equilibrium magnetization curve.

Moreover, we have compared our theory to experimental data on the cuprates. For this purpose, we have introduced a dimensionless lower critical field $\mathcal{H}_{c1}$

$$\mathcal{H}_{c1} = \frac{4\pi\mu_0 H_{c1}}{\Phi_0},$$

where $\Phi_0 = h/(2e)$ is the superconducting flux quantum. According to our theory in the underdoped regime where $s \ll 1$, $\mathcal{H}_{c1}$ scales as $1/s$, whereas $\mathcal{H}_{c1}$ is more or less constant within the conventional theory. In Ref. [2], we have presented two data sets on underdoped cuprates which show that $\mathcal{H}_{c1}$ diverges as the superconductor-normal metal boundary is approached. This experimental result can not be explained by the conventional theory, but it is completely consistent with our modified GL theory [1,2].

The most interesting prediction of the modified GL theory [1] is a sharp peak of the magnetic field in the vicinity of the vortex core for $s < 1$. Small angle neutron scattering (SANS) and muon spin rotation (μSR) are modern powerful tools for probing the microscopic distribution of the magnetic field. They have been successfully applied in determining the magnetic phase diagram, characteristic length scales, and internal magnetic field in the vortex state of type-II superconductors [3,4].

The aim of this paper is to demonstrate that SANS and μSR techniques can be used to test the predictions of the modified GL

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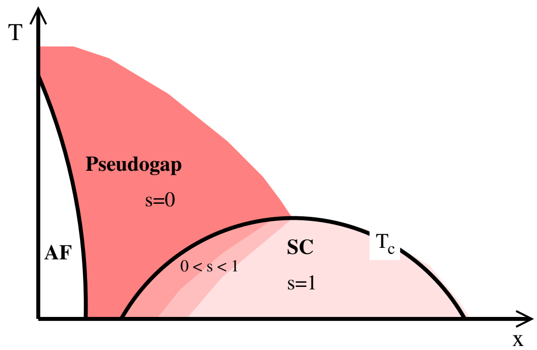


Fig. 1. Schematic phase diagram of the cuprates in the doping vs. temperature plane. We postulate that the ratio $s = \xi_\perp/\xi$ smoothly evolves between unity deep inside the superconducting region and zero in the pseudogap "phase".

theory [1,2]. In particular, in what follows we calculate the magnetic field profile in a triangular vortex lattice and we show how to use SANS and μSR to measure the parameter s of our theory.

2. Predictions of modified GL theory

It is worth mentioning that the Abrikosov lattice solution of the conventional GL equations cannot be easily modified to the case with $s \neq 1$ due to nonlinear terms in the modified GL equations. In Ref. [1], therefore we have constructed an approximate vortex-lattice solution of the Wigner–Seitz type. In this approach we have replaced the unit cell of the vortex lattice by a circular disk with radius d and inside the disk we have constructed radially symmetric solutions to the modified GL equations.

Note that the vortex lattice solution of Ref. [1] is exact in the vicinity of the vortex cores but it is not quite accurate in the area between the vortices. In order to remove this drawback, in the present work we combine the solution of Ref. [1] with the London solution at the edge of the unit cell. This will be achieved by the formula

$$B(\mathbf{r}) = f \cdot B_{WS} + (1-f) \cdot B_L, \qquad (1)$$

where B_{WS} represents solution of the Wigner–Seitz type and B_L represents solution of the London type. f is a mixing function which is unity for $|\mathbf{r}| < r_{min}$ and zero for $|\mathbf{r}| > r_{max}$. r_{min} and r_{max} are chosen in such a way that the function $B(\mathbf{r})$ be smooth in the vortex lattice, and each vortex carries one flux quantum.

All calculations presented in this paper were performed for ξ = 30 Å, λ = 1500 Å and two values of the average magnetic field $\overline{B}$. $\overline{B}$ is related to the radius of the Wigner–Seitz disk by the formula $\overline{B} = \phi_0/\pi d^2$. Our choice $d = 7\xi$ and $d = 5\xi$ corresponds to $\overline{B} = 1.44$ T and $\overline{B} = 2.83$ T, respectively. In Fig. 2, we plot the resulting magnetic field profile in the triangular vortex lattice for $d = 7\xi$.

2.1. Small angle neutron scattering

The scattered intensity of the neutron beam in a SANS experiment is directly related to the Fourier transform of the magnetic field in the vortex lattice. The Fourier components are given by [5]:

$$B_{\mathbf{q}} = \frac{1}{S}\int d^2\mathbf{r}\, B(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}}, \qquad (2)$$

where the wave-vectors $\mathbf{q}$ run over the points of the two-dimensional reciprocal lattice. Integration is taken over the magnetic unit cell with area $S = \pi d^2$. In Fig. 3, we plot $B_{\mathbf{q}}$ as a function of the magnitude $|\mathbf{q}|$ of the scattering vector. In Ref. [1] we have shown that in the vicinity of the vortex core, i.e. for $r \to 0$, the magnetic field is given as $B(\mathbf{r}) = B_{max} - \alpha r^{2s}$, where B_{max} and α are constants. Making use of this formula we obtain an analytic expression $B_q \propto q^{-(2s+2)}$ valid in the short-wavelength limit. This result, shown in the inset to Fig. 3 as a dashed line, is in agreement with our numerical results.

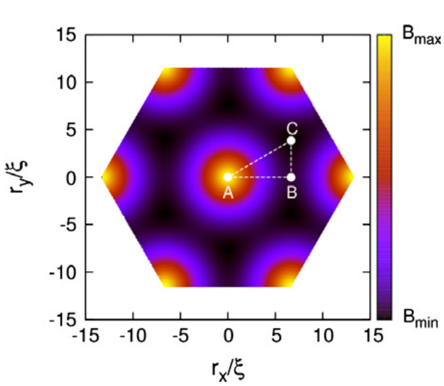
This shows that, at least in principle, measurements of the higher-order reflections of the neutron beam as a function of the reciprocal vector can therefore be used to verify the modified GL theory and the formula $B_q \propto q^{-(2s+2)}$ enables us to directly obtain the value of the parameter s. In the literature, we have found measurements of this type for borocarbide superconductors [4] and for classical type-II superconductors [6]. Unfortunately, we are not aware of similar studies on the cuprate superconductors.

2.2. Muon spin rotation

As shown in Ref. [7], monitoring the μSR signal as a function of time, one can observe the time dependence of the muon spin polarization vector. The Fourier transform of the polarization vector directly gives the magnetic field distribution $P(B)$, the probability that at an arbitrarily chosen point in the sample one finds the magnetic field B [8]. In the vortex lattice, $P(B)$ is given by the formula

$$P(B) = \frac{1}{S}\int d^2\mathbf{r}\, \delta(B - B(\mathbf{r})), \qquad (3)$$

where integration is to be taken over the magnetic unit cell with area S and δ is the Dirac delta function.



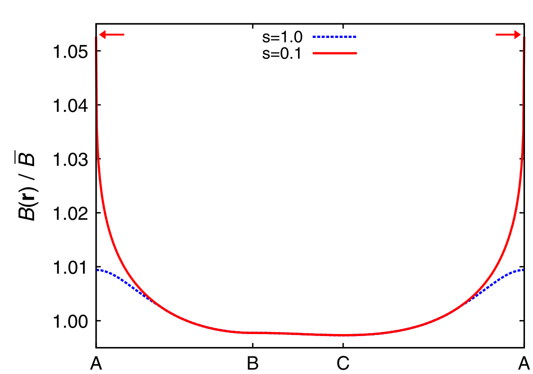


Fig. 2. *Left panel*: Magnetic field profile in the triangular vortex lattice for $s = 1$ and $d/\xi = 7$. The points A, B and C represent the vortex core center, the saddle point, and the global minimum of the magnetic field distribution, respectively. *Right panel*: Magnetic field along the path A–B–C for $d/\xi = 7$ and two values of s. Arrows indicate the maximum value of the magnetic field for $s = 0.1$. The most interesting feature is the sharp peak in the vicinity of the vortex core for $s < 1$.