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Superconductivity in a Fermi liquid from repulsive interactions: The role of electron–phonon interaction

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ABSTRACT

Based on our previously proven theorem that the interaction between a pair of quasiparticles in the normal Fermi liquid has an opposite sign to the interaction between particles, we consider pair correlation between a pair of quasiparticles when the interaction between particles is repulsive. For the convenience of statements, we have presented in this article once again the proof of the theorem in terms of an exact equation for the thermodynamic potential due to interaction between particles and based on the Green's function method. Further, we have derived the Landau expansion of the thermodynamic potentials in terms of the variation of the quasiparticle distribution function. We have also derived the expansion of the thermodynamic potential in terms of the variation of an exact single particle (not quasiparticles), these derivations lead to the relationship between the interaction function for two quasiparticles and the interaction energy between two particles as shown.

According to the proven theorem the interaction between a pair of quasiparticles is attractive in this case, the pairing – Cooper's pairing between a pair of quasiparticles is possible. We solve the Bethe–Salpeter type equation for paring of two quasiparticles when both interactions – the Coulomb repulsive and electron–phonon interaction are present. We show that the electron–phonon interaction, in fact, leads to the pair breaking effect, in contrast to the common belief that electron–phonon interaction is the main mechanism for Cooper's pair formation. We have calculated the transition temperature and the isotope effect on the transition temperature in terms of the Bethe–Salpeter equation.

1. Introduction

Since the discovery of high-temperature superconductivity in cuprates [1–3] and later in fullerides [4], magnesium diborides [5] and recently in iron-based pnictides [6–8] has been made numerous attempts to explain superconductivity in terms of the Coulomb repulsive model. All models of high-temperature superconductivity are based on boson-mediated exchange models involving different bosonic excitations rather than phonons. Thus, all the models of high-temperature superconductivity are in line with the original BCS model of superconductivity just replacing the phonons with some other type of bosonic excitations. Most recognizable model is the spin wave fluctuation model which has been applied to cuprates [9] and recently discovered iron-based pnictides [10]. In the spin-fluctuation model the exchange by spin fluctuation between a pair of electrons leads to the strong repulsive interaction as was first emphasized by Berk and Schrieffer as a pair breaking mechanism for metals showing strong ferromagnetic correlation. In the spin-fluctuation model the pairing occurs as a result of the sign change in the gap function connecting different points of the Brillouin zone. The spin-fluctuation model leads to the *d*-wave pairing in cuprates [9,11] and *s*± pairing in iron based pnictides [12]. All of high-temperature superconductivity models are in the area of strong electron correlation when weak-coupling models fail to operate. The area of strong electron correlation usually relies on the Fermi liquid theory where the properties of Fermi system are described in terms of quasiparticles. Even exotic models like RVB [13] and anyon models [14] still use concept of quasiparticles different from Fermi liquid theory. The Fermi liquid theory concept has been introduced by Landau to explain unusual properties of liquid ^{3}He in the normal state.

Landau's theory of Fermi liquids [15–19] is viewed as the most successful theory of modern physics, a theory whose importance goes well beyond its phenomenology. The central concepts of the Fermi liquid theory are the quasiparticle concept and the interaction energy between two quasiparticles. A quasiparticle is defined as an elementary excitation of a Fermi liquid with Fermi distribution similar to the spectrum of an ideal Fermi gas, whose effective

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mass is different from the mass of the particles forming Fermi liquid, but with the same spin and charge as a regular particle. The interaction energy between two quasiparticles is defined as the second order variational derivative of the ground state energy with respect to the quasiparticles distribution function. From the fundamental concepts and in terms of Green's function we have derived the effective renormalized and fully dressed interaction between two quasiparticles. We have also derived the relationship between the interactions of two quasiparticles and two particles, which is similar to the Ward's identity in many-particle theory. We show that the interaction between quasiparticles has opposite sign to the interaction between particles forming Fermi liquid. Based on the proven theorem, we consider Cooper's pairing between two quasiparticles due to the repulsive (Coulomb) interaction in terms of the Bethe–Salpeter equation. We have also examined the isotope effect.

2. Derivation of Landau interaction function between quasiparticles

We consider the thermodynamic potential instead of the ground state energy. The thermodynamic potential per unit volume of a system of interacting particles with spin ½ can be written in terms of an exact single-particle Green function in the well-known form [19–21]

$$\Omega = -i\int_{t\to+0}\left[\frac{\mathbf{p}^2}{2m} - \mu + \Sigma_\sigma(p)\right]G_\sigma(p)e^{i\omega t}\frac{d^4p}{(2\pi)^4}, \tag{1}$$

where in deriving Eq. (1) we have used the Dyson equation: $G_\sigma^{-1}(p) = G_{0\sigma}^{-1}(p) - \Sigma_\sigma(p)$, where $\Sigma_\sigma(p)$ is the exact single-particle self-energy part, $G_\sigma(p)$ is the exact single-particle Green's function of the system of interacting particles, with $G_{0\sigma}(p) = G_{0\sigma}(\mathbf{p},\omega) = [\omega - \mathbf{p}^2/2m + \mu + i\delta sign\omega]^{-1}$ being the free-particle Green function, μ is the chemical potential, while σ denotes particle's spin projection along the z axis. The integration in Eq. (1) assumes the summation over spin variable σ. We will keep spin indices because the Landau quasiparticle interaction function depends on the spin variables. Going to the conventional form for the Green's function by splitting the self-energy part into the real and imaginary parts and introducing the Fermi liquid renormalization factor as

$$Z_\sigma^{-1}(\mathbf{p},\omega) = 1 - \frac{\mathrm{Re}\Sigma_\sigma(\mathbf{p},\omega) - \Sigma_\sigma(\mathbf{p},0)}{\omega}, \tag{2}$$

In terms of the quasiparticle distribution function, the single-particle (quasiparticle) Green's function can be written in the well-known form [20,21]

$$G_\sigma(\mathbf{p},\omega) = Z_\sigma(\mathbf{p},\omega)\left[\frac{1-n_\sigma(\mathbf{p})}{\omega - \xi_\sigma(\mathbf{p}) + i\gamma_\sigma(\mathbf{p},\omega)} + \frac{n_\sigma(\mathbf{p})}{\omega - \xi_\sigma(\mathbf{p}) - i\gamma_\sigma(\mathbf{p},\omega)}\right], \tag{3}$$

where $\xi_\sigma(\mathbf{p}) = Z_\sigma(\mathbf{p},0)(\mathbf{p}^2/2m - \mu + \Sigma_\sigma(\mathbf{p},0)) \cong v_F(|\mathbf{p}| - p_F)$ is the quasiparticle energy with $\mu = p_F^2/2m + \Sigma_\sigma(p_F,0)$ being the chemical potential, which defines the Fermi momentum p_{Fs}, and $v_F = v_{0F}(m/m^*)$ being the quasiparticle Fermi velocity, while $v_{0F} = p_F/m$ is the particle Fermi velocity, where $m/m^* = Z(\mathbf{p},0)(1+\alpha(\mathbf{p}))$ defines the quasiparticle mass with $\alpha(\mathbf{p}) = (\Sigma(\mathbf{p},0) - \Sigma(p_F,0))/v_{0F}(|\mathbf{p}| - p_F)$ being the velocity renormalization factor; and $\gamma_\sigma(\mathbf{p},\omega) = -Z_\sigma(\mathbf{p},\omega)\,\mathrm{Im}\,\Sigma_\sigma(\mathbf{p},\omega)$ is the quasiparticle damping that vanishes on the Fermi surface ($\gamma_\sigma(\mathbf{p},0) = 0$); as well as $n_\sigma(\mathbf{p})$ is the quasiparticle distribution function that is assumed to be the usual Fermi distribution function. Eq. (3) implies that $Z(\mathbf{p},\omega)$ and $\alpha(\mathbf{p})$ are slowly varying function of ω and $\mathbf{p}$, respectively, across the Fermi surface. Using Eq. (3) in Eq. (1) and performing integration in Eq. (1) over ω, the fourth component of the four dimensional momentum, Eq. (2) for the thermodynamic potential takes on the form [21]

$$\begin{aligned}\Omega &= \sum_{\mathbf{p},\sigma}\left[\frac{\mathbf{p}^2}{2m} - \mu + \Sigma_\sigma(\mathbf{p},\xi_\sigma(\mathbf{p}))\right]Z_\sigma(\mathbf{p},\xi_\sigma(\mathbf{p}))n_\sigma(\mathbf{p}) \\ &= \sum_{\mathbf{p},\sigma}\tilde{\xi}_\sigma(\mathbf{p})n_\sigma(\mathbf{p}),\end{aligned} \tag{4}$$

where we have introduced $\tilde{\xi}_\sigma(\mathbf{p})$, the redefined quasiparticle energy which, as shown below, is the same as $\xi_\sigma(\mathbf{p})$ used and defined in Eq. (3), and other quantities are defined as above. It is not surprising that the thermodynamic potential of the system of interacting fermions is exactly the same as the thermodynamic potential of the free Fermi gas, but, in fact, Eq. (4) is the thermodynamic potential of the Fermi gas of interacting quasiparticles – *the Fermi liquid*. We can expand the thermodynamic potential Eq. (4) in terms of the variation of the quasiparticle distribution function $\delta n_\sigma(\mathbf{p})$, following the Landau's Fermi liquid theory, and equate each term of the expansion with the corresponding terms in the Landau expansion. Thus, we have [15–18]

$$\begin{aligned}\Omega &= \Omega_0 + \sum_{\mathbf{p}\sigma}\frac{\delta\Omega}{\delta n_\sigma(\mathbf{p})}\delta n_\sigma(\mathbf{p}) \\ &\quad + \frac{1}{2}\sum_{\mathbf{p}\sigma,\mathbf{p}'\sigma'}\frac{\delta^2\Omega}{\delta n_\sigma(\mathbf{p})\delta n_{\sigma'}(\mathbf{p}')}\delta n_\sigma(\mathbf{p})\delta n_{\sigma'}(\mathbf{p}'),\end{aligned} \tag{5}$$

where Ω_0 is the thermodynamic potential on the ground state. We can define that

$$\begin{aligned}\xi_\sigma^{(0)}(\mathbf{p}) &= \frac{\delta\Omega}{\delta n_\sigma(\mathbf{p})} \\ &= Z_\sigma(\mathbf{p},\xi_\sigma^{(0)}(\mathbf{p}))\left[\frac{\mathbf{p}^2}{2m} - \mu + \Sigma_\sigma(\mathbf{p},\xi_\sigma^{(0)}(\mathbf{p})) + \Sigma_\sigma^{Hq}(\mathbf{p},\xi_\sigma^{(0)}(\mathbf{p}))\right],\end{aligned} \tag{6}$$

where

$$\Sigma_\sigma^{Hq}(\mathbf{p},\xi_\sigma^{(0)}(\mathbf{p})) = \frac{1}{Z_\sigma\left(\mathbf{p},\xi_\sigma^{(0)}(\mathbf{p})\right)}\sum_{\mathbf{p}'\sigma'}f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')n_{\sigma'}(\mathbf{p}'), \tag{7}$$

is the Hartree self-energy part for quasiparticles; and $f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')$ is defined as the quasiparticle interaction energy given by

$$f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}') = Z_\sigma(\mathbf{p},\xi_\sigma(\mathbf{p}))\frac{\delta\Sigma_\sigma(\mathbf{p},\xi_\sigma(\mathbf{p}))}{\delta n_{\sigma'}(\mathbf{p}')}. \tag{8}$$

In derivation of Eq. (7) we have used the property of the quasiparticle interaction function as follows: $f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}') = f_{\sigma'\sigma}(\mathbf{p}',\mathbf{p})$. The second order variational derivative of Ω, the thermodynamic potential, with respect to the variational derivative of the quasiparticle distribution function $\delta n_\sigma(\mathbf{p})$, with the use of Eqs. (6)–(8) can be written as

$$\begin{aligned}\frac{\delta^2\Omega}{\delta n_\sigma(\mathbf{p})\delta n_{\sigma'}(\mathbf{p}')} &= Z_\sigma(\mathbf{p},\xi_\sigma(\mathbf{p}))\frac{\Sigma_\sigma(\mathbf{p},\xi_\sigma(\mathbf{p}))}{\delta n_{\sigma'}(\mathbf{p}')} + Z_\sigma(\mathbf{p},\xi_\sigma(\mathbf{p})) \\ &\quad\times\frac{\Sigma_\sigma^{Hq}(\mathbf{p},\xi_\sigma(\mathbf{p}))}{\delta n_{\sigma'}(\mathbf{p}')} \\ &= f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}') + f_{\sigma\sigma'}^H(\mathbf{p},\mathbf{p}'),\end{aligned} \tag{9}$$

where $f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')$ is defined in Eq. (8), and $f_{\sigma\sigma'}^H(\mathbf{p},\mathbf{p}')$ is the quasiparticle interaction function in the Hartree channel and is defined similarly to Eq. (8) for $f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')$ as follows

$$f_{\sigma\sigma'}^H(\mathbf{p},\mathbf{p}') = Z_\sigma(\mathbf{p},\xi_\sigma(\mathbf{p}))\frac{\delta\Sigma_\sigma^{Hq}(\mathbf{p},\xi_\sigma(\mathbf{p}))}{\delta n_{\sigma'}(\mathbf{p}')}, \tag{10}$$

where $\Sigma_\sigma^{Hq}(\mathbf{p},\xi_\sigma(\mathbf{p}))$ is the Hartree self-energy part for quasiparticles defined in Eq. (7). Eqs. (6)–(8) define the quasiparticle ground state energy $\xi_\sigma^{(0)}(\mathbf{p})$ and the interaction energy between two quasiparticles $f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')$, respectively. The last term in Eq. (6) in the right hand side in the square brackets $\Sigma_\sigma^{Hq}\left(\mathbf{p},\xi_\sigma^{(0)}(\mathbf{p})\right)$ is the Hartree term which