Physica C 495 (2013) 5–9

Contents lists available at ScienceDirect

Physica C

journal homepage: www.elsevier.com/locate/physc

Vortex contribution to the critical temperature oscillations in hybrid superconductor-ferromagnet coaxial cylinders

B. Krunavakarn [a,*], S. Yoksan [b]

[a] Department of Physics, Faculty of Science, Burapha University, Chonburi 20131, Thailand
[b] Department of Physics, Faculty of Science, Srinakharinwirot University, Bangkok 10110, Thailand

ARTICLE INFO

Article history:
Received 7 June 2013
Accepted 12 July 2013
Available online 28 July 2013

Keywords:
Vortex
Triplet superconductivity
Ferromagnetism
Proximity effect
Critical temperature

ABSTRACT

The s-wave triplet Usadel equations which include the odd frequency triplet condensate are employed to study the oscillatory behavior of the superconducting critical temperature T_c in multiply connected hybrid systems e.g., a ferromagnetic core surrounded by a superconducting shell. This configuration allows us to treat the angular momentum number L of the Cooper pair as the vorticity parameter. The spiral exchange field in the ferromagnet characterized by a spiral wave vector Q, which rotates in the z-plane and its direction varies within an in-plane of the ferromagnetic core. The induced superconductivity in the ferromagnet core is controlled by the electrical contact at the boundary for which we are interested in the switching of the superconducting states among the various vorticity numbers. The peculiar superconducting states with $L \neq 0$ bear resemblance to the pi-phase state in the bilayer systems. We demonstrate the switching phenomena by solving the Usadel equations together with the self-consistent order parameter subject to the Kupriyanov-Lukichev boundary conditions at the contact surface to obtain the secular equation in the multimode method to determine the oscillatory behavior of T_c in the ferromagnet core. The Abrikosov–Gorkov like-formula is also obtained within the single mode approximation.

1. Introduction

There are some similarity between the vortex states and the proximity effect in hybrid superconductor (SC) and ferromagnet (FM) structures [1,2]. The former one is associated with the orbital effect and causes the oscillation of the critical temperature T_c of a multiply connected superconducting sample [3] in an applied magnetic field H which is known as the Little-Parks effect. The latter one is responsible for the interplay between the ferromagnetic moment and the superconducting order parameter. The important feature of the proximity effect manifests itself in the oscillating or reentrant behavior of T_c as a function of a ferromagnetic layer thickness in FM/SC structures [4].

The characters of both effects differ in nature, namely, for the vortex state the interaction of the Cooper pairs with the orbital magnetic field resulting in the switching between various vortex states characterized by different winding numbers, whereas, for the proximity effect the interaction of the Cooper pairs with the exchange field causes the penetration of Cooper pairs inside the FM layer. However, though they are different, the common situation is therefore the switching phenomenon of the Cooper pair wavefunction states. Therefore the interplay between the orbital and exchange effects is of particular interest to determine whether the exchange interaction can stimulate the superconducting states with a non-zero vorticity in the absence of an applied magnetic field.

Our attention is paid to the behavior of the superconducting critical temperature with different vorticities in multiply connected domains in which we choose a sample to be a ferromagnetic core covered by a superconducting shell. Within this selected geometry the angular momentum L of the Cooper pair wavefunction can represent the vorticity parameter. Thus the switching states between different vorticities will display the non-monotonic T_c behavior.

Our purpose is to study the possibility of the occurrence of vortex states in hybrid SC/FM coaxial cylinders by investigating the behavior of the superconducting critical temperatures with different vorticities. This work may be considered as extension studies of Samokhvalov et al. [5,6] here we take into account the spiral magnetization inside the FM core and use the Usadel equations that contain both singlet and triplet pair amplitudes [7]. We determine T_c of the proximity system by solving the secular equation exactly. This paper is organized as follows. In Section 2, the formulation of the generalized Usadel equations with boundary conditions are presented. In Section 3, we demonstrate the exact calculations of T_c in the multimode method. Finally, conclusions are presented in Section 4.

* Corresponding author. Tel.: +66 38102222.
E-mail address: boonlit@buu.ac.th (B. Krunavakarn).

0921-4534/$ - see front matter
http://dx.doi.org/10.1016/j.physc.2013.07.006

2. Model and formulation

In the dirty-limit condition the elastic electron-scattering time τ, the critical temperature T_c, and the exchange length h satisfy the conditions $T_c\tau \ll 1$ and $h\tau \ll 1$ so an appropriate method to calculate T_c of the system is based on the Usadel equations [8]. It is well known that the Usadel approach is suitable to apply to the proximity effect system when the layers are spatially separated, and the interplay between superconductivity and ferromagnetic ordering is feasible. In each layer the exchange field exists only in the ferromagnet whereas the superconducting order parameter is localized in the superconductor. However, below T_c the pair potential can leak into the ferromagnet through the contact surface. Hence the superconducting properties are modified by the proximity effect.

Near the second order phase transition, the generalized Usadel equations (in the absence of the external magnetic field) take the linearized form [7,9]

$$(\omega - \frac{D}{2}\nabla^2)F_s(\boldsymbol{r},\omega) + i\boldsymbol{I}(\boldsymbol{r})\cdot\boldsymbol{F}_t(\boldsymbol{r},\omega) = \Delta(\boldsymbol{r}), \tag{1}$$

$$(\omega - \frac{D}{2}\nabla^2)\boldsymbol{F}_t(\boldsymbol{r},\omega) + i\boldsymbol{I}(\boldsymbol{r})F_s(\boldsymbol{r},\omega) = 0, \tag{2}$$

where F_s and $\boldsymbol{F}_t = (F_x, F_y, F_z)$ are, respectively, the scalar spin singlet and the vector spin triplet pair amplitudes. The Matsubara frequencies $\omega = (2n + 1)\pi T$, with $n = 0, \pm1, \pm2, \ldots$, and the parameter D is the diffusion constant which labeled either D_s in SC or D_f in FM.

The order parameter Δ and the pair amplitude F_s in the spin singlet s-wave state are related self-consistently by the equation

$$\Delta(\boldsymbol{r}) = 2\pi T\lambda\sum_{\omega>0}F_s(\boldsymbol{r},\omega), \tag{3}$$

where λ is the dimensionless BCS coupling constant which exists only in the SC layer.

The vector exchange field $\boldsymbol{I}$ is chosen to be the spiral type which rotates in an in-plane of the FM layer with a spiral wave vector Q,

$$\boldsymbol{I} = h(\cos Qz, \sin Qz, 0). \tag{4}$$

By means of the proximity effect the pair amplitude functions are connected by the boundary conditions [10,11]

$$\xi_s\nabla F^s = \gamma\xi_f\nabla F^f, \tag{5}$$

$$F^s = F^f - \gamma_b\xi_f\hat{n}\cdot\nabla F^f, \tag{6}$$

at the interface and

$$\nabla F^{s,f} = 0, \tag{7}$$

at the vacuum surface. The superscripts s and f on F are labeled to distinguish SC from FM layers. $\xi_{s,f} = \sqrt{D_{s,f}/2\pi T_{c0}}$ is the corresponding diffusion length, where T_{c0} is the isolated superconducting critical temperature. The parameter γ is the resistivity mismatch between the two layers in contact where usually $\gamma < 1$, as a result of the pair leakage from SC to FM. γ_b is the boundary resistance which represents the pair jumping from SC to FM in the direction outward normal to the interface $\hat{n}$.

The geometry of the FM/SC system, see Fig. 1, consists of a ferromagnetic cylinder $r < d_f$ covered by a superconducting shell $d_f < r < d_f + d_s$. This coaxial cylinder has a length c. In such geometry the angular momentum number L serves as the vorticity parameter, we introduce [5]

$$\Delta(\boldsymbol{r}) = e^{iL\theta}\Delta(r,z), \quad F(\boldsymbol{r},\omega) = e^{iL\theta}F(r,z,\omega) \tag{8}$$

where $L = 0$ corresponds to the non-vortex state and $L \geqslant 1$ represents the contribution of the vortex energy.

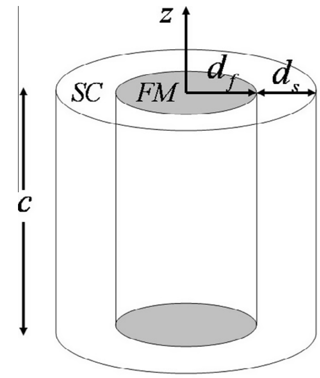

Fig. 1. Geometry of the FM/SC system: a ferromagnetic core of radius d_f surrounded by a superconducting shell has width d_s. Here c is a length of the cylinder.

When the spiral exchange field $\boldsymbol{I}(\boldsymbol{r})$ is substituted into the Usadel equations, the triplet component F_z can be disregarded because it is not coupled to F_s. Then by introducing the Champel-Eschrig transformation [7]

$$F_x \pm iF_y = 2e^{\pm iQz}F_\pm, \tag{9}$$

we arrive at the effective Usadel equations

$$\left[\omega - \frac{D}{2}\nabla^2\right]F_s(\boldsymbol{r},\omega) + ih[F_+(\boldsymbol{r},\omega) + F_-(\boldsymbol{r},\omega)] = \Delta(\boldsymbol{r}), \tag{10}$$

$$\left[\omega - \frac{D}{2}(\nabla^2 \pm 2iQ\partial_z - Q^2)\right]F_\pm(\boldsymbol{r},\omega) + \frac{ih}{2}F_s(\boldsymbol{r},\omega) = 0. \tag{11}$$

Within the transformation (9), the boundary conditions still possess the canonical form as follows

$$\xi_s\partial_r F_s^s = WF_s^s, \tag{12}$$

at the FM/SC interface $r = d_f$, where

$$W = \left.\frac{\gamma\xi_f\partial_r F_s^f}{F_s^f + \gamma_b\xi_f\partial_r F_s^f}\right|_{r=d_f}, \tag{13}$$

is the boundary function. Other boundary conditions are

$$\partial_r F_s^s = 0, \quad \partial_r F_\pm^s = 0, \tag{14}$$

at the vacuum/SC surface $r = d_f + d_s$, and

$$\gamma\xi_f\partial_r F_\pm^f = \frac{\xi_s\partial_r F_\pm^s}{F_\pm^s}\left[F_\pm^f + \gamma_b\xi_f\partial_r F_\pm^f\right], \tag{15}$$

at $r = d_f$. Therefore Eqs. (12)–(15) are the boundary conditions at the lateral sides ($r = d_f$, $d_f + d_s$), there are also the boundary conditions at the top $z = c$, and bottom $z = 0$

$$\partial_z F_{s,\pm}^{s,f}|_{z=0,c} = 0. \tag{16}$$

Having introduced the vorticity parameter L, we can show that the eigenfunctions $\cos(q_m z)$, where $q_m = m\pi/c$ with $m = 0, \pm1, \ldots$, satisfy the Neumann type boundary conditions at $z = 0$ and $z = c$, then the z-dependent of $\Delta(\boldsymbol{r})$ and $F(\boldsymbol{r}, \omega)$ can be expanded as

$$\Delta(\boldsymbol{r}) = e^{iL\theta}\sum_{m=-\infty}^{\infty}\cos(q_m z)\Delta(r,q_m), \tag{17}$$

$$F(\boldsymbol{r},\omega) = e^{iL\theta}\sum_{m=-\infty}^{\infty}\cos(q_m z)F(r,q_m,\omega). \tag{18}$$

The effective Usadel Eqs. (10) and (11) subject to the boundary condition (16) can be further simplified by using the expansion (17) and (18)