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Effect of impurities with retarded interaction with quasiparticles upon critical temperature of s-wave superconductor

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ABSTRACT

Generalization of a disordered metal's theory has been proposed when scattering of quasiparticles by impurities is caused with a retarded interaction. It was shown that in this case Anderson's theorem was violated in the sense that embedding of the impurities in s-wave superconductor increases its critical temperature. The increasing depends on parameters of the metal, impurities and their concentration. At a specific relation between the parameters the critical temperature of the dirty superconductor can essentially exceed critical temperature of pure one up to room temperature. Thus the impurities catalyze superconductivity in an originally low-temperature superconductor.

1. Introduction

Real superconductors are disordered metal containing various kinds of impurities and lattice defects. Quasiparticles scatter by these objects that influences upon superconductive properties of a metal – critical temperature, gap, critical fields and currents change. It is well known impurities are two kinds – magnetic and nonmagnetic. The magnetic scattering differently acts on components of Cooper pair (for singlet pairing), with the result that its decay takes place. Superconductive state is unstable regard to embedding of magnetic impurities - critical temperature decreases that is accompanied by effect of gapless superconductivity. In a case of nonmagnetic impurities an ordinary potential scattering acts on both electrons of a cooper pair equally. Therefore the pair survives. Hence, *superconductive state is stable regard to introduction of nonmagnetic impurities – a gap and critical temperature of a superconductor do not change*. This statement is Anderson's theorem – T_C and $\Delta(T)$ of an isotropic *s*-wave superconductor do not depend on presence of nonmagnetic impurities [1–4]. This phenomenon is result of the gap function Δ and the energy parameter ε being renormalized equally [3]. In a case of anisotropic *s*-pairing a weak suppression of T_C by disorder takes place [5,6]. For *d*-wave pairing the nonmagnetic impurities destroy superconductivity like magnetic impurities [5–8]. It should be noted that phonons with lower energies than temperature of the electron gas are perceived by the electrons as static impurities. Hence the thermal phonons have no effect on the critical temperature of a s-wave superconductor that is described by Eliashberg's equations [32]. However in high-T_C oxides the thermal excitations can break Cooper pairs [33] because *d*-wave pairing takes place. Besides a superconductive state is unstable regard to introduction of nonmagnetic impurities if the gap is an odd function of $k - k_F$ [10]. If pairing of electrons with nonretarded interaction takes place then T_C quickly decreases with an increase of disorder [11].

The disorder can influence upon phonon and electron specter in materials. It results to both increase and decrease of T_C. Experiments in superconductive metal showed suppression of T_C by a sufficiently strong disorder [12–15]. The strong disorder means that a free length l is such that $\frac{1}{k_F l} \approx 1$ or $\varepsilon_F \tau \approx 1$, where $\tau = l/v_F$ is a mean free time. For weak superconductors as Al or In a dependence of T_C on a disorder $\frac{1}{k_F l}$ has a maximum, but finally the strong disorder leads to decrease of T_C always [25], strong superconductors (Pb, Hg) have not this maximum [16–18]. In the experiments a total pattern was found: collapse of superconducting state takes place near Anderson's transition metal–insulator, that is when $\frac{1}{k_F l} \gtrsim 1$. It should be notice that superconduction appears in amorphus films of Bi, Ga, Be ($T_C \sim 10$ K) just when these materials are not superconductors in a crystal state [30]. In such systems superconducting is result of intensification of electron–phonon interaction by disorder. Nowadays universal mechanisms of influence of a disorder upon T_C are unknown. Several mechanisms of degradation of T_C were supposed: a growth of Coulomb pseudopotential μ^* [19–21], influence of the disorder upon a density of states on Fermi

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surface $\nu(\xi)$ [22,23] – evolution of Altshuler–Aronov singularity [3,24] into "Coulomb gap". We will not consider these phenomenons as violation of Anderson's theorem because they have other nature and we will consider a weak disorder $\frac{1/l}{k_F} \ll 1$ that is far from a metal–insulator transition.

Introduction of nonmagnetic impurities in a superconductor is widely used in a practice: the impurities essentially increase a critical current and a critical magnetic field but do not change critical temperature at the same time. Our problem is to find such impurities which violates Anderson's theorem in the direction of essentiality increasing of the critical temperature T_C. Obviously it is matter of nonmagnetic impurities in a three-dimensional superconductor with s-wave order parameter Δ. The impurities have to play a role of a catalyst of superconductivity in an originally low-temperature superconductor. It should be notice that in an article [36] it was shown that in s-wave superconductors small amounts of nonmagnetic impurities can increase the transition temperature. However the correction is of the order of T_C/E_F, and this effect is result from local variations of the gap function near impurity sites. Thus the effect is not violation of Anderson's theorem.

Nowadays a theory of disordered systems has been well developed for elastic scattering of conduction electrons by impurities [3,4,9,24,31]. In a total case the scattering can be inelastic that is an impurity's potential depends on time $\upsilon(t)$. In this case to develop a perturbation theory (to unlink and to sum a diagram series) is impossible. In a Section 2 it will be shown that in a special case of *retarded* interaction with impurities the perturbation theory can be built and a theory of disordered systems can be generalized. In a Section 3 it will be shown these impurities violates Anderson's theorem in the direction of increase of T_C. A change of the critical temperature depends on both impurities' parameters and electronic parameters of a metal matrix. At specific combinations of the parameters the critical temperature can essentially exceed critical temperature of a pure metal and has values up to room temperature.

2. Retarded interaction of conduction electrons with impurities

Let us consider an electron moving in a field created by N scatterers (impurities) which are placed in a random manner with concentration $\rho = \frac{N}{V}$. A random distribution of the impurities in a space corresponds to a distribution function: $P(\mathbf{R}_j) = V^{-N}$. Let a potential of an impurity is a function of coordinates and time: $\upsilon(\mathbf{r} - \mathbf{R}_j, t)$, where $\mathbf{R}_j$ is an impurity's coordinate $\mathbf{r}$ is an electron's coordinate. A total potential created by the impurities is:

$$V(\mathbf{r},t) = \sum_{j=1}^{N} \upsilon(\mathbf{r} - \mathbf{R}_j, t) = \frac{1}{V}\sum_{\mathbf{q}}\sum_{j} \upsilon(\mathbf{q},t) e^{i\mathbf{q}(\mathbf{r}-\mathbf{R}_j)}, \quad (1)$$

where $\upsilon(\mathbf{q},t)$ is Fourier transform of the potential, $\upsilon(-\mathbf{q},t) = \upsilon^*(\mathbf{q},t)$. In most cases the potential can be considered as point, so that $\upsilon(\mathbf{q}) \approx \upsilon = \int \upsilon(\mathbf{r})d\mathbf{r}$. Thus the system is spatially inhomogeneous and nonconservative.

Considering the potential as weak a perturbation theory can be constructed writing the secondary quantized interaction Hamiltonian of an electron with the field (1) as $H_{int} = \int d\mathbf{r}\psi^+(\mathbf{r})V(\mathbf{r},t)\psi(\mathbf{r})$. Then a perturbation series for an electron's propagator has a view:

$$G(1,1') = G_0(1,1') + \int d2 G_0(1,2)V(2)G_0(2,1') + \int d2d3 G_0(1,2)V(2)G_0(2,3)V(3)G_0(3,1') + \ldots, \quad (2)$$

where $1 \equiv (\mathbf{r},t)$, $1' \equiv (\mathbf{r}',t')$. The averaging over an ensemble of samples with all possible positions of impurities recovers spatial homogeneity of a system. In a representation of secondary quantization the averaging operation over a disorder has a form [35]:

$$G(x,x') = -i\frac{\left\langle \widehat{T}\psi^+(x)\psi(x')\widehat{U}\right\rangle_0}{\left\langle \widehat{U}\right\rangle_0} \longrightarrow \langle G(x,x')\rangle = -i\left\langle \frac{\left\langle \widehat{T}\psi^+(x)\psi(x')\widehat{U}\right\rangle_0}{\left\langle \widehat{U}\right\rangle_0}\right\rangle_{\text{disorder}}, \quad (3)$$

where $\widehat{U}$ is an evolution operator, $\langle\ldots\rangle_0$ is done over a ground state of Fermi system and a lattice (in the numerator and the denominator separately). The averaging over the disorder is done as follows: at first the propagator is calculated at the given disorder, and only then the averaging $\langle\ldots\rangle$ is done (the whole fraction is averaged). At averaging of the series (2) $G(\mathbf{r},\mathbf{r}',t) \to \langle G(\mathbf{r},\mathbf{r}',t)\rangle$ in a limit $\rho \to \infty$, $\upsilon^2 \to 0$, $\rho\upsilon^2 = \text{const}$ the averages appear with factorized correlators:

$$\langle V(\mathbf{r}_1)V(\mathbf{r}_2)\rangle = \rho\upsilon^2\delta(\mathbf{r}_1 - \mathbf{r}_2), \quad \langle V(1)\rangle = 0, \quad \langle V(1)V(2)V(3)\rangle = 0, \ldots$$
$$\langle V(1)V(2)V(3)V(4)\rangle = \langle V(1)V(2)\rangle\langle V(3)V(4)\rangle + \langle V(1)V(4)\rangle\langle V(2)V(3)\rangle + \ldots, \quad (4)$$

that corresponds to motion of an electron in Gauss random field with a white noise correlator. Then an electron's propagator is determined with a sum of diagrams shown in Fig. 1 (a diagrammatic techniques of averaging over disorder [3]). In an analytic form we have (we use rules of diagrammatic techniques presented in [26]):

$$iG(\mathbf{k},t_1,t_2) = iG_0(\mathbf{k},t_2 - t_1) + \int dt' \int dt'' iG_0(\mathbf{k},t' - t_1)iG_0(\mathbf{k},t_2 - t'') \cdot \rho\int \frac{d^3q}{(2\pi)^3}(-i)\upsilon(\mathbf{q},t')iG_0(\mathbf{k}-\mathbf{q},t''-t')(-i)\upsilon(-\mathbf{q},t'') + \ldots \quad (5)$$

Here $G_0(\mathbf{k},t_2 - t_1)$ is a free electron's propagator depending on a time difference (a pure system is conservative), G is a dressed electron's propagator. Since potential of an impurity is a function of a *point of time* $\upsilon = \upsilon(\mathbf{q},t)$, then diagrams of higher orders cannot be uncoupled, and the series (5) cannot be summed (energy is not conserved). The series can be summed partially in the following cases only. In the first case an impurity's potential does not depend on time $\upsilon = \upsilon(\mathbf{q})$. It means that an electron scatters elastically by impurities. It is well described by the disordered system theory [3,4,24,31]. Necessary to us concepts of the theory are presented in Appendix A.

In this article we propose another case when the series (5) can be uncoupled and summed partially. The case is when an impurity's potential is a function of a time difference between consecutive scatterings. That is an interaction of electrons with impurities is retarded (advanced). In the first approximations a dependence of the scattering potential on a time difference can be considered as harmonic:

$$\upsilon(\mathbf{q},t')\upsilon(-\mathbf{q},t'') = \begin{Bmatrix} \upsilon(\mathbf{q})\upsilon(-\mathbf{q})e^{-i\omega_0(t''-t')}; & \text{for} \quad t'' > t' \\ \upsilon(\mathbf{q})\upsilon(-\mathbf{q})e^{i\omega_0(t''-t')}; & \text{for} \quad t'' < t' \end{Bmatrix}, \quad (6)$$

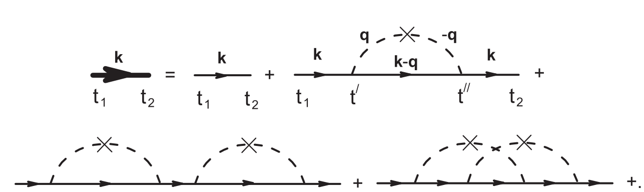

Fig. 1. The diagram expansion of an averaged Green function $G(k,t)$ in a random field (4). Dotted lines with daggers means action of the averaged summarized field of impurities in a momentum space – a transfer of an intermediate momentum $\mathbf{q}$.