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Fermion quartets in a two-band model of superconductivity

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ABSTRACT

A two-band superconducting fermion system is investigated. Apart from an on-site two-fermion attraction acting in the wider band an on-site four-fermion attraction is introduced. The four-fermion potential binds four electrons from both of the bands residing on a lattice site into a fermion quartet. Additionally, the local Coulomb repulsion is involved in the narrower band. There exist two order parameters in the system: a conventional BCS-type gap and a quartet one. The ground state properties such as the dependence of these order parameters on the Coulomb repulsion and the four-fermion attraction are examined. Furthermore, the average numbers of electrons constituting the system as functions of the interactions are displayed as well. It is shown that quartet superconductivity is very sensitive to the Coulomb repulsion and when subject to this interaction quartets disappear for quite small values of that. Finally, the response to an external magnetic field is studied. This reveals a quite complicated behavior of the system because in general two electric currents can flow through a specimen but there exist conditions under which one of them disappears. Such a behavior is determined by the dependence of both currents on the strength of the Coulomb repulsion and the four-fermion attraction. Two scenarios with $d_{x^2-y^2}$-wave pairing are investigated on an initial stage as well.

1. Introduction

Multi-band electron systems have been a subject of intensive studies for a few last decades. This was begun in 1959 year when Moskalenko [1] and Suhl et al. [2] published the first papers on a two-band model of superconductivity. A multitude of papers on different multi-band scenarios have been appeared since that time. These systems turned out to be especially intriguing due to a simple fact – they can describe the variety of physical systems and their phases such as superconductivity, magnetism or the Kondo effect. By their efficiency in producing the richness of physics, one is justified when saying that they are much closer to real systems than simple one-band models (the one-band Hubbard model is frequently included in those). Eminent examples are the periodic Anderson model [3–7], the Kondo lattice one that is derived from the first one [8], the three-band Hubbard model [9], the d-p model [10] and finally the two-band Hubbard-Holstein model [11].

On account of that the incorporation of a four-fermion attraction into a multi-band Hamiltonian describing unconventional superconductivity seems to be especially provoking. The idea of introducing the four-fermion attraction to the description of superconductivity appeared in 1996 [12]. It consisted in the completion of the conventional one-band BCS Hamiltonian with such an interaction. Fermion quartets resulting from this interaction were made up of electrons from one band. The spectrum and thermodynamics of the full Hamiltonian were investigated in [13,14]. For example, it was shown that the specific heat jump at the critical temperature was slightly enhanced with respect to the BCS value. It was made by making use of perturbation theory with the four-fermion attraction as perturbation of the BCS ground state. This approach had a defect consisting in neglecting the effect of quartets on Cooper pairs. This led to the lack of the dependence of a Cooper pair gap on a quartet gap. This can affect the thermodynamic functions, e.g., the specific heat of the mixture of Cooper pairs and quartets and as a consequence the height of the jump of one.

Unfortunately, the analysis of the one-band model comprising these two types of electron clusters is quite tough in view of the complex general structure of eigenvalues and the form of equations for the energetic gaps. That is why two approaches have been applied so far. The first one has already been described above. The other consisted in neglecting the bandwidth as being small in comparison with the BCS interaction [15]. That approximation led to a significant simplification of all the integrands in the theory and enabled to investigate the thermodynamics for an arbitrarily

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chosen value of the coupling constant of the four-fermion attraction.

In this paper a new two-band model of superconductivity is investigated. It is assumed that the conventional on-site BCS attraction is present in the wider band while the weak on-site Coulomb repulsion acts in the narrower one. On the other hand one can consider the same system with the Coulomb repulsion replaced by the attraction. In such a case one deals with superconductivity in both of the bands. However, it is the incorporation of an on-site four-fermion attraction comprising pairs of electrons from two bands that is completely new. If one admits the existence of two Cooper pairs in two bands residing on a lattice site then it will be quite natural to consider a possibility that they can interact and form a quartet. Superconducting fermion systems in the strong coupling regime may display the possibility of the formation of quartets. In [16] a Fröhlich type transformation was applied to a phonon-electron Hamiltonian and this revealed the presence of fermion quartets in higher order terms of the expansion. This procedure used to a two-band phonon-electron Hamiltonian would reveal them for sure. Next, there exists a record of experimental observations. The strongest argument for quartets as real entities comes from the surveys of magnetic flux quanta. It is widely known that if half-$\frac{h}{2e}$ magnetic flux quanta appear among usual ones it points to the existence of quartets in an investigated system.

Such quanta have been observed in bicrystalline $YBa_2Cu_3O_{7-\delta}$ films [17,18] and in Sr_2RuO_4 [18,19]. Furthermore, it is important that beside these quanta signals pointing to existence of sextets and more complex clusters appeared. In connection to that Aligia et al. [20,21] presented a theoretical description of this experiment in terms of quartet formation. This was based on a boson-lattice model transformed by means of a boson-spin transformation into an anisotropic spin-$\frac{1}{2}$ model. Moreover, Stankowski et al. investigated the magnetically modulated microwave absorption of the $YBa_2Cu_3O_7 - Pb(Sc_{0.5}Ta_{0.5})O_3$ superconducting composite (with $(0.5;0.5)$ composition) and found evidence pointing to the presence of fermion pairs and quartets in this material [22]. Of course, there were some indications of the presence of fermion quartets earlier. In 1993 Schneider and Keller [23] measured the relationship between the critical temperature and zero temperature condensate density of some cuprates and Chevrel-phases superconductors. They noticed that the experimental data for $YBa_2Cu_3O_{6.602}$, for example, point to the behavior of a dilute Bose gas. As a result they suggested Bose condensation of weakly interacting fermion pairs as a mechanism of the transition from the normal to the superconducting state. Moreover, a discovery of Bunkov et al. [24] points to the presence of fermion quartets in ^{3}He. Their work was devoted to the problem of the influence of spatial disorder on the order parameter in superfluid ^{3}He. The authors, quoting Volovik [25], suggested that unusual spectra of ^{3}He in aerogel could be explained by a process in which impurities tend to destroy the anisotropic correlations of the order parameter, while correlations of higher symmetry can survive (e.g. four-particle correlations). Regarding four-particle correlations Koh pointed out that the inclusion of them into the Gorkov decoupling scheme leads to more stable low temperature phase in a superconductor than in the conventional BCS system [26].

This paper is organized as follows. In Section 2 the model is introduced. Section 3 describes the diagonalization of a reduced Hamiltonian resulting from the application of some approximations to the initial one and gives the structure of eigenvectors and eigenvalues. Section 4 deals with properties of the ground state. In turn, Section 5 is devoted to the problem of the response of the investigated system to an external magnetic field at zero temperature. In the last section we consider two scenarios in which $d_{x^2-y^2}$-wave pairing is introduced. This is done by making use of perturbation theory with the four-fermion attraction serving as perturbation.

2. The model

We are dealing with the following Hamiltonian

$$\begin{aligned} H = &\sum_{i\neq j,\sigma} \tilde{t}^c_{ij} c^*_{i\sigma} c_{j\sigma} + \sum_{i\neq j,\sigma} \tilde{t}^d_{ij} d^*_{i\sigma} d_{j\sigma} + (\widetilde{E}^d - \mu) \sum_{i\sigma} n^d_{i\sigma} \\ &+ (\widetilde{E}^c - \mu) \sum_{i\sigma} n^c_{i\sigma} - \widetilde{U}^c \sum_i n^c_{i+} n^c_{i-} + \widetilde{U}^d \sum_i n^d_{i+} n^d_{i-} \\ &+ \widetilde{V} \sum_{i\sigma} (c^*_{i\sigma} d_{i\sigma} + d^*_{i\sigma} c_{i\sigma}) + \widetilde{U}^{cd} \sum_{i\sigma\sigma'} n^d_{i\sigma} n^c_{i\sigma'}, \end{aligned} \tag{2.1}$$

completed with the following interaction term

$$H' = -U_4 \sum_i n^c_{i+} n^c_{i-} n^d_{i+} n^d_{i-}, \tag{2.2}$$

with

$$\tilde{t}^c_{ij} = t^c_{ij} e^{-\left(\frac{g^c}{\hbar\omega}\right)^2}, \quad \tilde{t}^d_{ij} = t^d_{ij} e^{-\left(\frac{g^d}{\hbar\omega}\right)^2}, \quad \widetilde{E}^c = E^c - \frac{g^{c2}}{\hbar\omega},$$
$$\widetilde{E}^d = E^d - \frac{g^{d2}}{\hbar\omega}, \quad \widetilde{V} = V e^{-\frac{1}{2}\left(\frac{g^c - g^d}{\hbar\omega}\right)^2}, \quad \widetilde{U}^c = U^c - 2\frac{g^{c2}}{\hbar\omega},$$
$$\widetilde{U}^d = U^d - 2\frac{g^{d2}}{\hbar\omega}, \quad \widetilde{U}^{cd} = U^{cd} - 2\frac{g^c g^d}{\hbar\omega}.$$

The derivation of Hamiltonian (2.1) from a model standing for an extension of the periodic Anderson model (PAM) was shifted to Appendix A. The extension consists in completing PAM with local phonon-electron interactions. Hamiltonian (2.1) is obtained via making use of a generalized Lang-Firsov transformation that eliminates the phonon-electron interactions to all orders from the Hamiltonian. The full Hamiltonian $H + H'$ describes a very complicated system of interacting c and d-electrons in a solid. $c^*_{i\sigma}$ and $c_{i\sigma}$ are creation and annihilation operators of an electron from the c-band (the wider one) whereas $d^*_{i\sigma}$and $d_{i\sigma}$ concern electrons from the narrower d one, respectively. σ denotes the spin of electrons and i and j refer to lattice sites. $n^c_{i\sigma}$ and $n^d_{i\sigma}$ are the number operators of both kinds of electrons. $\widetilde{E}^c$ and $\widetilde{E}^d$ refer to the renormalized due to phonons site energies of both kinds of electrons, respectively. μ is the chemical potential. The parameters $\widetilde{U}^c, \widetilde{U}^d$ and $\widetilde{U}^{cd}$ are the effective coupling constants of the local Coulomb interactions between electrons from the same bands and the different ones. The interaction between c-electrons is assumed to be attractive leading in this way to appearance of superconductivity in the system. The second one can be treated arbitrarily, however it is assumed here that d-electrons repel each other. In the case of the attractive potential acting between d-electrons a new pairing channel appears that leads to superconductivity as well. The third one is usually taken on as repulsive and is neglected here. Therefore Hamiltonian (2.1) can be substantiated by the existence of strong local interactions between phonons and electrons in some materials [27,28]. Such interactions lead to the formation of polarons which in turn can attract each other. These interacting polarons can be bound into so-called bipolarons. Thus, superconductivity can be caused by on-site interactions of local phonons with local electrons and its emerging depends really on the magnitude of Coulomb repulsions and electron–phonon interactions. One could ask the question about one-particle hybridization of two bands but there is the serious argument that the strong coupling of electrons to lattice deformations results in the very pronounced reduction of