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Coherent control of the gap function in superconductors on the nanoscale

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ABSTRACT

We propose a nonequilibrium mechanism of the gap function enhancement in nanoscale superconductors. The mechanism is based on coherent control and localization of the spatial distribution of Bogoliubov quasiparticles on the scale much less than the wavelength of the control field.

1. Introduction

A superconducting transition is characterized by the gap in the excitation spectrum and the critical transition temperature T_c, which both depend on the effective phonon mediated attraction between electrons and the density of states in the system near the Fermi level. The search for new materials with an enhanced phonon interaction mechanism or a higher density of states to provide a higher T_c is regarded as the "Holy Grail" of modern condensed matter physics [1]. Here we propose mechanism of a nonequilibrium enhancement of the local gap value, which is not based on direct control of the effective interaction between electrons, but rather relays on control of the local density of states through the spatial localization of the quasiparticle density in the system. The local gap function $\Delta(\mathbf{r},t)$ at any given position $\mathbf{r}$ and time moment t depends on the product of the local amplitudes of quasi-electron and quasi-hole (Bogoliubov) excitations $\sum_i v_i^*(\mathbf{r},t)u_i(\mathbf{r},t)$ [2]. If the external control field is chosen in such way that it drives a significant amount of the quasiparticles to be localized at a certain time moment t_0 in the vicinity of $\mathbf{r}_0$, this can lead to an enhancement of the local gap value $\Delta(\mathbf{r}_0,t_0)$. Therefore, the gap function can be enhanced locally in the target volume rather than in the whole system, and this enhancement is achieved along with the decreasing of the quasiparticles density and the local gap function elsewhere, as it shown in Fig. 1. Similar ideas motivated the studies on spatial localization of atomic and molecular wavepackets on the scale much less than wavelength of the external control field [3]. In the present work we consider coherent control of quasiparticle subbands, which leads to spatial localization of quasiparticles and the local gap enhancement. The local gap value is approximately proportional to the locally defined critical transition temperature $T_c(x_0,t_0) \propto \Delta(x_0,t_0)$.

The suggested coherent control mechanism should be contrasted with the known Eliashberg mechanism of the transition temperature enhancement in superconductors [4]. In the latter case a time-dependent electromagnetic field creates a nonequilibrium distribution of quasiparticles in the superconductor that may lead to unoccupied states at the Fermi surface at the gap edge, leading to an increase of the gap. However, the Eliashberg mechanism does not result in a *spatially localized* enhancement of the quasiparticle density. The Eliashberg mechanism is a relatively weak effect, resulting in increase of the transition temperature T_c by an order of 1%. This mechanism was observed and extensively studied in the cases of electromagnetic, acoustic or tunneling nonequilibrium processes [5].

2. Setup of the control problem

To illustrate the proposed enhancement mechanism let us consider the interaction of a thin film nanoscale superconductor with a time dependent control field. A nonequilibrium state of the system is described using time-dependent Bogoliubov-de Gennes (TDBdG) equations for inhomogeneous systems [2]:

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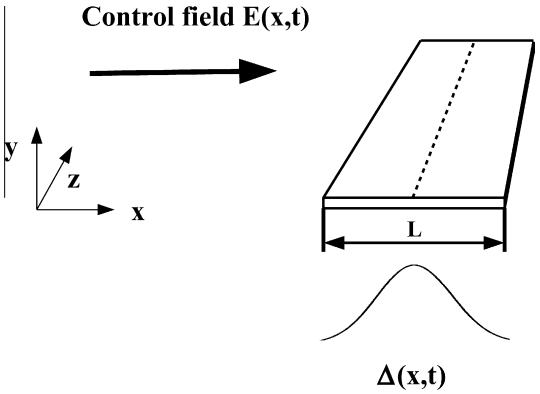

Fig. 1. Optimal control of a nanoscale superconducting nanoribbon using external field $E(x,t)$. Note the local enhancement of the gap function in the center of the nanoribbon, at the expenses of the gap function decrease near the edges.

$$i\hbar\frac{\partial u_n(\mathbf{r},t)}{\partial t} = Hu_n(\mathbf{r},t) + \Delta(\mathbf{r},t)v_n(\mathbf{r},t), \tag{1}$$
$$i\hbar\frac{\partial v_n(\mathbf{r},t)}{\partial t} = -Hv_n(\mathbf{r},t) + \Delta^*(\mathbf{r},t)u_n(\mathbf{r},t),$$

with $H = H_0 + U_H(\mathbf{r},t) + V_{ext}(\mathbf{r},t) - \mu, H_0$ defined by

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + W_{trap}(\mathbf{r}). \tag{2}$$

Here m is the electron mass, μ is the chemical potential, $W_{trap}(\mathbf{r})$ is the effective trapping potential, $V_{ext}(\mathbf{r},t)$ is the external control potential. $U_H(\mathbf{r},t)$ is the Hartree-like mean field local potential, given by

$$U_H(\mathbf{r},t) = -D_0\sum_n[|u_n(\mathbf{r},t)|^2 f_n + |v_n(\mathbf{r},t)|^2(1-f_n)], \tag{3}$$

where f_n is the quasiparticle occupation number determined by the Fermi distribution function. The quantity of our interest, time dependent local gap function $\Delta(\mathbf{r},t)$, which is also called the pairing potential, is given by

$$\Delta(\mathbf{r},t) = D_0\sum_n u_n(\mathbf{r},t)v_n^*(\mathbf{r},t)(1-2f_n). \tag{4}$$

Here D_0 is the effective electron attraction coefficient. The summation in Eq. (4) is performed over the eigenstates with the single electron energies within the Debye window around the Fermi level, $[E_F-\omega_D, E_F+\omega_D$, where ω_D is the cutoff (Debye) energy.

Let us consider a system with the nanoribbon geometry shown in Fig. 1, and set the trapping potential $W_{trap}(x)$ to be a square well of the length L with infinite walls. The external control field $E(x,t) = -\nabla V_{ext}(x,t)$ is assumed to be linearly polarized along the x direction, perpendicular to the nanoribbon. In the limit of the control field wavelength much bigger than the system's size, the control field $E(t)$ has a constant amplitude, and the control potential is a linear function of the coordinate: $V_{ext}(x,t) = -E(t)x$.

Low dimensional systems similar to the shown in Fig. 1 can be realized, for example, using modern graphene-based materials. Recently lithium deposition was suggested to achieve superconductivity in graphene [6]. About at the same time an efficient chemical method was developed to produce graphene nanoribbons with the width, which can be varied in a wide range, as small as 10 nm [7]. The thickness of the superconducting system should not be too small to prevent the phase slip phenomena [8], which is not included in our mean field description. One may also consider an experiment with quasi one dimensional superconductors, such as a nanowire or a bundle of doped carbon nanotubes [9,10]. Making analogy with the Eliashberg effect [11], one may expect that a nonequilibrium perturbation of the system may increase the effective critical temperature, as well as the instantaneous gap function.

For the setup shown in Fig. 1 the quasiparticle amplitudes can be represented as $v_n(\mathbf{r},t) = v_n(x,t)e^{-ik_z z}\sin(\pi y/d), u_n(\mathbf{r},t) = v_n(x,t)e^{-ik_z z}\sin(\pi y/d)$, where the index n counts the quasiparticle band, and the thickness of the nanoribbon d is small enough so one can neglect the excitations in this dimension. The TDBdG equations Eq. (1) become:

$$i\hbar\frac{\partial u_n(x,t)}{\partial t} = H_{1D}u_n(x,t) + \Delta(x,t)v_n(x,t), \tag{5}$$
$$i\hbar\frac{\partial v_n(x,t)}{\partial t} = -H_{1D}v_n(x,t) + \Delta^*(x,t)u_n(x,t),$$

where

$$H_{1D} = -\frac{d^2}{dx^2} + \frac{\hbar^2k_z^2}{2m} + \frac{\hbar^2\pi^2}{2md^2} + U_H(x,t) - \mu + V_{ext}(x,t). \tag{6}$$

At zero temperature $T=0$ the gap function becomes

$$\Delta(x,t) = D_0\sum_n u_n(x,t)v_n^*(x,t), \tag{7}$$

and the Hartree-like mean field local potential takes form of

$$U_H(x,t) = -D_0\sum_n|v_n(x,t)|^2. \tag{8}$$

3. Optimal control of the gap function

Optimal control problem can be formulated as a search for an optimal external field $V_{ext}(x,t)$, which maximizes the average value of the gap function $\langle\Delta\rangle$ in the target region $[x_0-\epsilon_0, x_0+\epsilon_0]$ over a given time interval $[T_f-t_0, T_f]$:

$$\langle\Delta\rangle = (2t_0\epsilon_0)^{-1}\int_{T_f-t_0}^{T_f}\int_{x_0-\epsilon_0}^{x_0+\epsilon_0}|\Delta(x,t)|\,dx\,dt. \tag{9}$$

In order to understand how is it possible to control the quantity Eq. (9), we do the following perturbative analysis. The TDBdG equations Eq. (5) can be approximately solved in the absence of the control field ($V(x,t)\equiv 0$), assuming constant gap function $\Delta(x)=\Delta_0$ and Hartree potential $U_H(x)=U_0$. The analytical solution in this case is

$$u_n(x,t)\approx\bar{u}_n\psi_n(x)e^{-i\frac{E_n t}{\hbar}}, v_n(x,t)\approx\bar{v}_n\psi_n(x)e^{-i\frac{E_n t}{\hbar}}, \tag{10}$$

with $\bar{u}_n^2 = \frac{1}{2}\left(1+\frac{\epsilon_n-\tilde{\mu}}{E_n}\right)$, $\bar{v}_n^2 = \frac{1}{2}\left(1-\frac{\epsilon_n-\tilde{\mu}}{E_n}\right)$, where $\tilde{\mu} = \mu - \frac{\hbar^2k_z^2}{2m} - \frac{\hbar^2\pi^2}{2md^2} - U_0$. The eigenenergies are $E_n = \sqrt{(\epsilon_n-\tilde{\mu})^2+\Delta_0^2}$, where ϵ_n and $\psi_n(x)$ are the eigenenergies and eigenfunctions for the stationary Schroedinger equation with the Hamiltonian Eq. (6). In the considered geometry the system consists of several subbands of quasiparticle amplitudes. For the chosen geometry the unperturbed wavefunctions are close to the analytical solution for noninteracting electrons in the infinite well potential: $\psi_n(x) = \frac{1}{\sqrt{L}}\sin(\pi nx/L)$. A control field that maximizes $\langle\Delta\rangle$ should drive at least one subband amplitude u_n, v_n from its initial state $u_n(x,0), v_n(x,0)$ to a spatially localized nonequilibrium state at the target time $u_n(x,T_f), v_n(x,T_f)$ (assuming $t_0 \ll T_f$), which will have an enhanced magnitude at the target region $[x_0-\epsilon_0, x_0+\epsilon_0]$. The region of the enhanced gap function is shown by the dashed line in Fig. 1.

Similar optimal control problem was solved analytically in the case of optimal squeezing of a wavepacket of an atom in an infinite well potential using multifrequency control field [12]. As an initial guess for the control field $V_{ext}(x,t)$ we choose one that drives the initial quasiparticle subband amplitude with the lowest energy $E_1, u_1(x,0), v_1(x,0)$ to a nonequilibrium spatially localized state. There is some freedom of choice of the target localized state.