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Superconducting frustration bit

Y. Tanaka *

National Institute of Advanced Industrial Science and Technology, AIST-Tsukuba-Central-2-3035918, 1-1-1 Umezono, Tsukuba, Ibaraki 305-8568, Japan

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ABSTRACT

A basic design is proposed for a classical bit element of a superconducting circuit that mimics a frustrated multiband superconductor and is composed of an array of π-Josephson junctions (π-junction). The phase shift of π provides the lowest energy for one π-junction, but neither a π nor a zero phase shift gives the lowest energy for an assembly of π-junctions. There are two chiral states that can be used to store one bit information. The energy scale for reading and writing to memory is of the same order as the junction energy, and is thus in the same order of the driving energy of the circuit. In addition, random access is also possible.

1. Introduction

The interband phase difference of a conventional multiband superconductor is typically locked at either π or zero. Recently, non-π and non-zero interband phase-locked states (fractional phase locking) have become frequently addressed [1–15]. Fractional phase locking was considered to be realized under the degenerate electronic band structure [1]. However, recent studies have shown that the key issue for fractional phase locking is the frustration introduced by positive interband interaction rather than from the degenerate band structure [2,4,5–7,11]. Strong intraband interaction to align the phase in the band is also necessary, otherwise the entropy invokes other states that have multiple nodes in the one band at a finite temperature so that the intraband phase eventually becomes unsettled [7,16,17].

The chiral state in the frustrated three-band superconductor is one example to realize fractional phase locking [1,4–6,11,12]. The arrangement of the order parameters is shown in Fig. 1. The positive interband interaction results in two states designated as the R- (anti-clockwise arrangement of order parameters in Fig. 1) and L-states (clockwise arrangement). The presence of these two degenerate states is desirable for an information bit.

Multicomponent superconductivity based on multiband superconductors is now under exploration by many researchers [1–38]. Although frustrated superconductivity based on the multiband superconductor has yet to be established, there are some successful examples to emulate the multiband superconductor using an artificial multilayer system [39–41]. The interband phase difference soliton has been emulated in these systems and fractional vortices were found. A new functional device may be inspired by introducing the recent development of the π-junction (in which a phase shift of π gives the minimum junction energy) composed of superconducting and magnetic layers into an artificial multiband superconductor [42–47]. The π-junction corresponds to the positive interband interaction in a frustrated multiband superconductor. A classical bit element can be designed using a multiband superconductor mimic composed of π-junctions. Such a new classical bit element has been awaited for application to a superconducting circuit.

In the context of a multiband superconductor, the interband phase difference is considered as a new gauge field. Thus, control of the fractionally phase locking state means control of this new gauge field [19,26,32–34]. In a multiband superconductor mimic composed of multilayers, this gauge field (interlayer phase difference) can be controlled by application of a current between the layers. Combining these principles, we propose here a basic design for a classical bit element using the π-junctions and artificial multilayers.

2. Scheme for the circuit design

In this analysis, we concentrate on a case in which the size of the device is smaller than the magnetic penetration depth, i.e., inductance (L) of the device and the phase shift due to external and self-induced magnetic fields are neglected. It is assumed that the phase shift occurs at the junctions and phase rotation outside the junctions is ignored. When the π-junctions are used, there is

* Tel.: +81 29 861 5720.
E-mail address: y.tanaka@aist.go.jp

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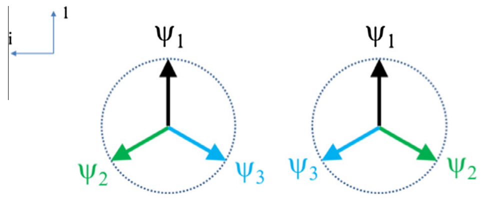

Fig. 1. Arrangement of the order parameters for three-component frustrated superconductivity based on the three-band superconductor. The arrow labeled ψ_i denotes the phase of i-th component in the complex plane, of which the axes are marked at the top left. The left and right arrangements designate the L- and R-states, respectively.

no need to introduce the magnetic field to architect a frustrated network, unlike the conventional frustration circuit composed of zero-junctions with the magnetic field [45,47] (In the conventional zero-junction network, a phase shift of zero gives the minimum coupling energy without the external magnetic field [48], and the magnetic field is necessary to achieve the frustration.). This is suitable to reduce the size of the memory device. As in a direct current superconducting quantum interference device (DC-SQUID), we also provide the over-damped situation to prevent the running state under the critical current. The operation of a single bit can then be analyzed using a standard "tilted washboard" potential, in which the effective potential of the junction describes the sum of the junction energy and the incline due to the external current [49].

Firstly, the Josephson junction energy and DC Josephson current are defined. Note that the sign of the parameters is slightly different from the notations of a standard text book [49]. However, this notation is convenient for analysis of the frustration bit. All of the junctions are composed of π-junctions to eliminate unexpected confusion. However, the frustrated situation can also be realized by substituting zero-junctions for some π-junctions.

The Josephson junction energy $E_{Junction}$, and the direction of the current I (Fig. 2) are defined as:

$$E_{Junction} = E_A \cos(\alpha), \tag{1-1}$$

$$I = I_A \sin(\alpha), \tag{1-2}$$

$$I_A = |\frac{2e}{\hbar}| E_A, \tag{1-3}$$

where $E_A > 0$ is a Josephson coupling constant, α is the phase difference in which the phase of the left part is subtracted from the phase of the right part in Fig. 2 [50], e is the elementary charge, and ħ is the Planck constant divided by 2π. In this scheme, the direction of pair flow is reversed against the increment of the phase due to the π-junction, whereas the direction of current flow is in the same direction along the increment of the phase. The tilted washboard potential U (in which the potential energy incline of the junction due to the current flow through the junction is considered) can be:

$$U(\alpha) = E_J \cos(\alpha) + |\frac{\hbar}{2e}| I\, \alpha. \tag{2}$$

For a network of Josephson junctions, the total potential U_{Total}, can be their sum:

$$U_{Total} = \sum E_{Ji} \cos(\alpha_i) + |\frac{\hbar}{2e}| I_i\, \alpha_i, \tag{3}$$

where i denotes the index of a junction. For a closed circuit where there is no external current $I_{ex} = 0$, $\sum |\frac{\hbar}{2e}| I_i\, \alpha_i = 0$ and $U_{Total} = \sum E_{J\,i} \cos(\alpha_i) = \sum E_{Junction\ i}$. We define $E_{Junction} \equiv \sum E_{Junction\ i}$ hereafter. When there is an external current from one inlet to another outlet, $\sum |\frac{\hbar}{2e}| I_i \alpha_i = |\frac{\hbar}{2e}| I_{ex} \alpha_{out-in}$. α_{out-in} is the phase difference between the outlet and the inlet. We then obtain

$$U_{Total} = \sum E_{Ji} \cos(\alpha_i) + |\frac{\hbar}{2e}| I_{ex}\ \alpha_{in-out}. \tag{4}$$

The main purpose of the analysis is to deduce the phase difference at the junction and between the inlet and outlet. The current conservation law (Kirchhoff's current law) is used with the single-valuedness of the superconducting order parameter, in which there is only one superconducting phase (modulo 2π) at any location (the phase consistency). The latter condition corresponds to Kirchhoff's second law (for the voltage) for the ordinary circuit.

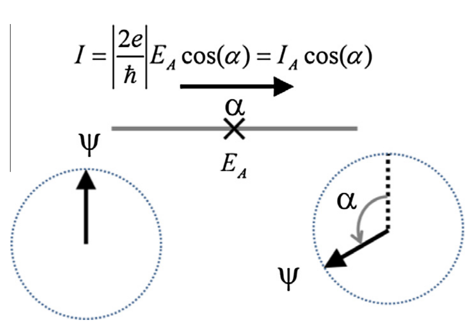

Fig. 2. Definition of the phase shift at a junction and the current flow. The superconducting phase is denoted by the arrows, as in Fig. 1.

3. Circuit composed of three banks and three junctions

Three superconducting banks connected by three π-junctions can emulate three component superconductivity based on a three-band superconductor. The basic circuit design is shown in Fig. 3. Let E_A, E_{B1}, and E_{B2} be the Josephson coupling constant, $E_{J\ i}$. A, B_1, and B_2 are used to denote the junctions hereafter. We assume $E_A < E_{B1} < E_{B2}$. The two expected degenerate chiral states are shown in Fig. 4(a1) and (b1). To realize the frustrated situation, we should be able to draw a triangle with $1/E_A$, $1/E_{B1}$, and $1/E_{B2}$ long sides, as shown in Fig. 4(a2) and (b2). Two triangles that are mirror images of each other correspond to the L- (Fig. 4(a2)) and R-states (Fig. 4(b2)). The exterior angle corresponds to the phase shift at each junction. Let α, β_1, and β_2 be the phase shift at junctions A, B_1, and B_2. The junction energy of this array is given by:

$$E_{Junction} = E_A \cos(\alpha) + E_{B1} \cos(\beta_1) + E_{B2} \cos(\beta_2). \tag{5}$$

Fig. 4 shows the order parameter of each bank, denoted as ψ_j with the index of the bank, j. The state of the circuit can be represented by the arrangement of ψ_j. By applying Kirchhoff's current law at the banks, the circuit should satisfy the following conditions:

$$I = I_A \sin(\alpha) + I_{B1} \cos(\beta_1), \tag{6-1}$$

$$I_{B1} \cos(\beta_1) = I_{B2} \cos(\beta_2). \tag{6-2}$$

The condition derived from the phase consistency is:

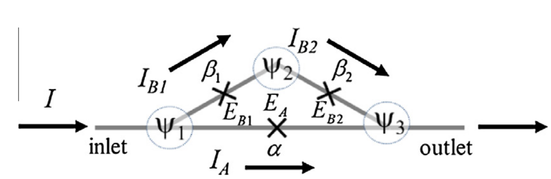

Fig. 3. Basic design of a frustration bit element composed of three banks, of which order parameters are denoted by ψ_1, ψ_2, and ψ_3, and three π-junctions. I denotes the external current, while I_A, I_{B1}, and I_{B2} are the currents through the junctions. E_A, E_{B1}, and E_{B2} are the coupling constants of the junctions. α, β_1, and β_2 are the phase shifts at the junctions.