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Photonic band gaps of a two-dimensional square lattice composed by superconducting hollow rods

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ABSTRACT

In this paper by means of the plane wave expansion method, we have calculated the photonic band structure of 2D photonic crystals consisting of high temperature superconducting hollow cylinders arranged in a square lattice. Band structures were obtained at low frequencies and assuming TM polarization of the incident wave, for different inner radii of the cylinders and for two different temperatures (5 K and 15 K), showing the tunability of photonic band gaps with respect to these parameters. Interesting features, such as the decreasing of cutoff frequency and separation of photonic modes were observed by increasing both the temperature and inner radius. Permittivity contrast and the difference between the inner and outer radius lead to the appearance of new band gaps when compared with the case of solid cylinders. These band gaps can be modulated by the width of the shell and temperature, which may be used for the development of novel optical devices.

1. Introduction

Since the publication of the works of Yablonovich [1] and John [2], photonic crystals (PCs) have been issue of investigation due to the tunability and mold of light passing through them. PCs are structures with periodicity in the permittivity and/or magnetic permeability of the materials of which they are composed. An important feature of PCs, is the inhibition of light propagation at some frequency ranges known as photonic band gaps (PBGs). Due to this, the application of PCs in optical devices, solar cells, LEDs and PC optical fibers have been considered [3–5].

PCs with different dielectric materials have been studied, where the dielectric permittivities are assumed constants. However, the possibility of using materials with dispersive dielectric permittivities has been achieved. In fact, many theoretical and experimental works on PCs made of metallic and high-temperature superconducting (HTSC) constituents, have been published [6–25]. Unlike conventional PCs, the PCs based on metallic and superconducting materials exhibit dispersive electromagnetic responses and high variability of PBG, besides operating in the terahertz frequency range. Particularly, works about PCs with HTSC constituents [13–24] has been proposed, due to the control of the PBG varying external parameters such as the system temperature and applied magnetic fields. Taking into account the negligible losses by dissipation below the critical temperature, the electromagnetic response of these materials, can be obtained assuming a non magnetic material ($\mu = 1$) and considering a frequency-dependent dielectric permittivity ($\epsilon(\omega)$), which is well represented in the context of the two-fluid model [15], where the dielectric permittivity can be reduced to that given by the Drude model for lossless metallic materials.

On the other hand, shells or hollow rods have been introduced in PCs, with the aim to obtain efficient tunable band gaps [26–30] by using different geometries of both the PC lattice and the hollow rods. In this kind of PC, the dielectric permittivity and the filling factor play an important role in the variation of the PBS, showing several differences with respect to PC with solid rods. One of them is the formation of new photonic modes and band gaps.

In this paper, we extend the idea of superconducting photonic crystals (SPCs), by considering cylindrical shell rods in two-dimensional square lattices. In order to obtain the photonic band structure (PBS), we use the plane wave expansion (PWE) method [31]. The shell rods, made of a high-temperature cuprate superconductor, are characterized by having inner and outer radii R_1 and R_2, respectively. Additionally, we assume TM polarization for the incident waves, in which the electric field is parallel to the anisotropy axis (c axis) of the superconducting material. The c axis is perpendicular to the CuO_2 planes (ab planes).

This work is organized as follows: the theoretical procedure to obtain the dielectric function is presented in Section 2, results and discussions of the photonic band structures are presented in Section 3, and conclusions in Section 4.

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2. Theoretical framework

We assume the electric field of the electromagnetic radiation parallel to the z axis ($E_z||c$). Besides, the shell rods with a dielectric permittivity $\epsilon_3(\omega)$, have an internal region characterized by a dielectric permittivity ϵ_2 and are embedded periodically in a background of air with permittivity ϵ_1. The unit cell and the three mentioned regions in the photonic crystal are shown in Fig. 1.

The dielectric permittivity along the c axis in cuprate superconductors can be obtained by using the two-fluid model, in which a fraction n_s of total electron density n is in the superconducting phase, and another one n_n is in the normal state, such that $n = n_s + n_n$ is conserved. In this model, the effective dielectric function is given by [15]

$$\epsilon_{eff}(\omega) = \epsilon_\infty \left(1 - \frac{\omega_{sp}^2}{\omega^2} - \frac{\omega_{np}^2}{\omega(\omega + i\gamma)}\right), \quad (1)$$

where

$$\omega_{sp} = \frac{c}{\lambda\sqrt{\epsilon_\infty}} \quad \text{and} \quad \omega_{np} = \sqrt{\frac{n_n e^2}{m\epsilon_0\epsilon_\infty}}, \quad (2)$$

are the plasma frequencies of superconducting and normal electrons along the c axis, respectively. ϵ_∞ is the dielectric permittivity at high frequencies of the superconducting material, c is the light speed, m the electron mass, γ a damping constant and λ is the London penetration depth, which in cuprate superconductors is related with the temperature by means of

$$\lambda(T) = \frac{\lambda_0}{\sqrt{1 - \frac{T}{T_c}}}, \quad (3)$$

where λ_0 is the penetration depth at zero temperature and T_c is the critical temperature of the superconducting material. Therefore, we can rewrite the superconducting plasma frequency as

$$\omega_{sp}(T) = \omega_{sp_0}\sqrt{1 - \frac{T}{T_c}}. \quad (4)$$

where ω_{sp_0} is the superconducting plasma frequency at zero temperature. We must note that Eq. (1) is valid for frequencies ω below the superconducting gap 2Δ, because at frequencies of the incident wave above that gap, the superconducting state can disappear. At sufficiently low temperatures, the dissipation effects associated with electrons in the normal state can be neglected, since at this temperature range it is satisfied the condition that $n_s \to n$, and the contribution to the dielectric permittivity of the electrons in the normal state vanishes. In that case, the dielectric permittivity takes the form

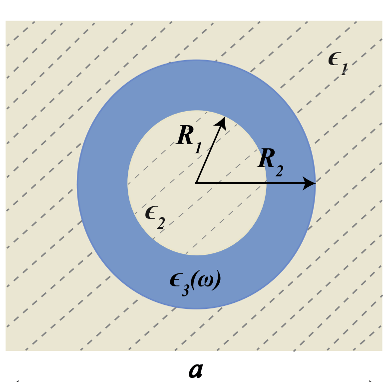

Fig. 1. Graphical representation for unit cell in the photonic crystal.

$$\epsilon_{eff}(\omega) = \epsilon_\infty \left(1 - \frac{\omega_{sp}^2}{\omega^2}\right), \quad (5)$$

which has the same structure as that of the lossless metallic materials in the Drude model. In order to obtain the photonic modes of two-dimensional SPC composed by cylindrical shell rods, we must solve the wave equation for the electric field, which is given by

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2}\epsilon(\mathbf{r})\mathbf{E}, \quad (6)$$

where $\epsilon(\vec{r}, \omega)$ is the dielectric permittivity of the superconducting photonic crystal in the xy plane, which involves both background and superconductor permittivities. By using the plane wave expansion (PWE) method for dispersive materials [33], we solve Eq. (6) as a eigenvalue problem, where ω^2/c^2 are the eigenvalues required for the photonic band structure. In order to solve Eq. (6) by means of the PWE method, we use the dielectric permittivity in the PC as $\epsilon(x, y) = \epsilon(x + R_x, y + R_y)$, where R_x and R_y are the lattice vectors. Therefore, we expand the dielectric permittivity in a Fourier series on the reciprocal lattice vectors G_x and G_y as

$$\epsilon(\mathbf{r}) = \sum_{\mathbf{G}} \chi(\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}}, \quad (7)$$

where

$$\chi(\mathbf{G}) = \frac{1}{\Omega}\int_{cell} \epsilon^{-1}(\mathbf{r}, \omega) e^{-i\mathbf{G}\cdot\mathbf{r}} d\mathbf{r}, \quad (8)$$

and Ω being the area of the unit cell. Here we have used $\vec{G} = (G_x, G_y)$ and $\vec{r} = (x, y)$ in our calculations. To calculate the Fourier coefficients for the dielectric permittivity in Eq. (7), we define the dielectric function of the system introducing the step function according with Fig. 1, we have

$$S_{rod}(\mathbf{r}) = \begin{cases} 1, & 0 < r < R_1, \\ 0, & \text{otherwise}, \end{cases} \quad (9)$$

and

$$S_{shell}(\mathbf{r}) = \begin{cases} 1, & R_1 < r < R_2, \\ 0, & \text{otherwise}. \end{cases} \quad (10)$$

In this way, the dielectric function for the square lattice is written as

$$\frac{1}{\epsilon(\mathbf{r})} = \frac{1}{\epsilon_1} + \left(\frac{1}{\epsilon_2} - \frac{1}{\epsilon_1}\right)S_{rod} + \left(\frac{1}{\epsilon_3} - \frac{1}{\epsilon_1}\right)S_{shell}. \quad (11)$$

By replacing Eq. (11) into Eq. (8), the Fourier coefficients for square lattice take the form

$$\chi(\mathbf{G}) = \begin{cases} \frac{1}{\epsilon_1} + \left(\frac{1}{\epsilon_2} - \frac{1}{\epsilon_1}\right)f_1 + \left(\frac{1}{\epsilon_3} - \frac{1}{\epsilon_1}\right)f_2, & \mathbf{G} = 0, \\ 2f_1\left(\frac{1}{\epsilon_2} - \frac{1}{\epsilon_1}\right)\frac{J_1(GR_1)}{GR_1} + \frac{2\pi}{G\Omega} \times & \\ \left(\frac{1}{\epsilon_3} - \frac{1}{\epsilon_1}\right)[R_2J_2(GR_2) - R_1J_1(GR_1)], & \mathbf{G} \neq 0, \end{cases} \quad (12)$$

where $J_1(x)$ is the Bessel function of the first kind, f_1 and f_2 are the filling fractions corresponding to solid and shell rods, respectively. Obviously, the filling fraction depends of lattice geometry. In fact, the filling fraction is the ratio between the occupied area by rods and the unit cell area. By using a similar procedure, the plasma frequency function can be obtained [33].

The periodic nature of the dielectric permittivity, allows us to expand the electric field in Fourier series as follows

$$E_z(\mathbf{r}, \omega) = \sum_{\mathbf{G}} E_z(\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}}. \quad (13)$$

By inserting Eqs. (7) and (13) into Eq. (6), we obtain the matrix related to the frequencies eigenvalue problem. Therefore, by solving the eigenvalue problem, we obtain the photonic band structure for SPC. In our case, we will show the PBS at the first Brillouin zone in the direction which connects high symmetry points.