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Temperature factor for magnetic instability conditions of type – II superconductors

V. Romanovskii *

NBIKS-Center, National Research Center 'Kurchatov Institute', Moscow 123182, Russia

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ABSTRACT

The macroscopic development of interrelated electrodynamics and thermal states taking place both before and after instability onset in type-II superconductors are studied using the critical state and the flux creep concepts. The physical mechanisms of the non-isothermal formation of the critical state are discussed solving the set of unsteady thermo-electrodynamics equations taking into consideration the unknown moving penetration boundary of the magnetic flux. To make it, the numerical method, which allows to study diffusion phenomena with unknown moving phase-two boundary, is developed. The corresponding non-isothermal flux jump criteria are written. It is proved for the first time that, first, the diffusion phenomena in superconductors have the fission-chain-reaction nature, second, the stability conditions, losses in superconductor and its stable overheating before instability onset are mutually dependent. The results are compared with those following from the existing magnetic instability theory, which does not take into consideration the stable temperature increase of superconductor before the instability onset. It is shown that errors of isothermal approximation are significant for modes closed to adiabatic ones. Therefore, the well-known adiabatic flux jump criterion limits the range of possible stable superconducting states since a correct determination of their stability states must take into account the thermal prehistory of the stable magnetic flux penetration. As a result, the calculation errors in the isothermal approximation will rise when the sweep rate of an external magnetic field or the size of the superconductor's cross-sectional area increase. The basic conclusions formulated in the framework of the critical state model are verified comparing the experimental results and the numerical analysis of the stability conditions and the temperature dynamics of the helicoid-type superconducting current-carrying element having real voltage–current characteristic.

On the whole, the non-isothermal stability conditions expand the existence of allowable stable superconducting states. The non-isothermal approximation permits also to link the theories of the losses, the magnetic instability and the thermal stabilization of superconductors, which are independently developed.

1. Introduction

The macroscopic electrodynamics of superconductors is one of the most important issues in their investigations. The results obtained are not only valuable to understanding the fundamental properties of superconducting materials but also permit the limits of their practical applications to be determined [1–4]. Therefore, the investigations of macroscopic formation mechanisms of superconducting states are important in basic studies of superconductors. They allow one to determine the margin of stable operating regimes of superconducting devices subjected to many external perturbations of different nature. As a result, they have made it possible to formulate the main principles lying on the basis of the theories of magnetic, current and thermal instabilities, which, in turn, lead to the conditions of retaining the superconductivity against electrodynamics and thermal perturbations.

In the past years many investigations have focused on experimental and theoretical studies of the magnetic instability problem [5–18]. Recently, the interest connected with this phenomenon increases again [19–26] since high-temperature superconductors have been widely used. However, the main conclusions of the existing magnetic instability theory were, as a rule, formulated basing on studies of the initial stage of thermal and electrodynamics states occurring inside the superconductor and resulting from the effect of infinitely small perturbation. Moreover, the finite

* Address: Department of Superconducting Magnets, Division of Superconductivity, NBIKS-Center, National Research Center 'Kurchatov Institute', Kurchatov's sq., 1, Moscow 123182, Russia. Tel.: +7 499 1967955; fax: +7 499 1965973.

E-mail address: vromanovskii@netscape.net

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temperature change in the superconductor before onset of instability depending on its operating mode was not taken into account in the framework of this approximation. At the same time, one of the main features of type-II superconductors is the dissipative processes occurring in them, which are due to thermal activated motion of vortices. Therefore, the thermal state of superconductor before instability may influence on its stability conditions. However, the existing investigations of the forming regimes of the superconducting states in the non-isothermal approximation [13–18,26] do not permit the general regularities determining the effect of stable variations in their temperature on the instability conditions to be formulated. To recognize this peculiarity, let us discuss early formulated conclusions, which deal with the magnetic instability conditions of type-II superconductors.

2. Existing flux jump conditions of type-II superconductors

As it follows from the critical state model, the magnetic instability does not exist [1–4], if the superconductor satisfies the so-called adiabatic stability condition

$$\beta = \mu_0 a^2 J_c^2(T_0, B_a)/[C(T_0)(T_{cB} - T_0)] < 3 \quad (1)$$

which has been obtained for a thermally isolated superconducting slab in the approximation $\Lambda = \mu_0 \lambda(T_0)/C(T_0)\rho_f \to 0$. Here, C is the specific heat capacity of the superconductor, λ is the coefficient of its heat conductivity, a is the half-thickness of a slab, T_0 is the coolant temperature, B_a is the external magnetic field, $J_c(T,B)$ is the critical current density of superconductor, T_{cB} is the critical temperature of the superconductor at B_a, ρ_f is the superconductor resistance in the resistive regime, μ_0 is the magnetic permeability of vacuum.

According to [1,3], the allowance for the final value of Λ leads to the corresponding correction in the right side of the criterion (1). In this case, it can be written as

$$\beta < \pi^2(1 + 2\sqrt{\Lambda})/4 \quad (2)$$

Criteria (1) and (2) obviously show that isothermal approximation was used to formulate the magnetic instability conditions. Indeed, the critical current density, specific heat capacity and heat conductivity coefficient are determined at coolant temperature T_0. In other words, these criteria describe the stability conditions of thermally isolated superconductor whose temperature is equal to the coolant temperature upon adiabatic penetration of magnetic flux in it.

1D-analysis of the magnetic instability conditions in the type-II superconductors taking into account the flux creep states described by the exponential equation of the voltage–current characteristic has been performed in [14–17,26] both for the low- and high-temperature superconductors. In particular, it was proposed a model in whose framework the instability criterion is written as follows [14]

$$\frac{1}{S}\int_S E ds > E_m = \frac{hp}{S}\frac{J_\delta}{J_c|\partial J_c/\partial T|} \quad (3)$$

using the E–J relation, which may generally written in the form $E = E_c \exp[J/J_\delta + (T_0 - T_{cB})/T_\delta]$. Here, h is the heat transfer coefficient, p is the cooled perimeter of the conductor, S is its cross-sectional area, T_δ and J_δ are the temperature and current creep parameters, E_c is the electric field criterion in J_c definition. Accordingly, the temperature of superconductor T_i before instability is small and equals

$$T_i = T_0 + J_\delta/|\partial J_c/\partial T| \quad (4)$$

when the current density J over the cross-sectional area of superconductor is constant.

The physical sense of condition (3) is obvious: the superconducting state is stable when the overage value of the electric field over its cross-sectional area is lower than the characteristic value E_m.

In [15] the low perturbation method was used for 1D-investigation of the superconducting state stability with allowance for variation in the back ground temperature of the superconductor prior to the magnetic instability onset. The corresponding flux jump field B_m was obtained. It satisfies the solution of equation

$$B_m^2 = \left(2\frac{h\mu_0^2 J_\delta}{\dot{B}} + 6\frac{h\mu_0^2 J_\delta}{\dot{B}}\sqrt{\frac{C\dot{B}B_p}{h\mu_0 J_\delta B_m}}\right)(T_{cB} - T_0) \quad (5)$$

Here, $\dot{B}$ is sweep rate of an external magnetic field and $B_p = \mu_0 a J_c|_{T=T_0}$ is the fully penetrated field.

According to [16], an allowable overheating of superconductor before the instability onset is low and equal to the temperature creep parameter T_δ, which satisfies the condition $T_\delta/(T_{cB} - T_0) \ll 1$. As a result of this feature, the following stability criterion

$$\int_S EJds \leqslant hpT_\delta \quad (6)$$

was formulated in [16] for a superconducting composite. Condition (6) has the following physical meaning: superconductivity is retained if the heat release in composite does not exceed the heat flux to the coolant at a constant permissible overheating, which is equal to T_δ regardless of the sweep rate, cooling conditions and the transverse dimension of a composite.

It is easy to find that criteria (3) and (5), which determine the first flux jump field in superconductors during flux creep, do not satisfy the limiting transition to criterion (1) at $h \to 0$ or $J_\delta \to 0$ because the flux jump field will be equal to zero in these transitions. As it follows from (6), superconductivity of composite conductor will be also inevitably destroyed by any infinitesimal perturbation at $h \to 0$ or $T_\delta \to 0$, i.e., the superconducting state of an adiabatically cooled superconductor with very steep transition E–J characteristic must be in principle unstable. In other words, the limiting transition to the critical state model ($J_\delta \to 0$ or $T_\delta \to 0$) is not possible in the framework of such approximations. As a result, the existing theory does not interconnect magnetic instability conditions obtained in the framework of the critical state model with ones taken place for superconductors with real E–J characteristics. Besides, theoretical approximations used do not give exact answer to the question on the dependence of an allowable overheating of superconductor before the onset of instability on the operating conditions and its influences on the stability conditions. For example, the conclusions made in [26] for Bi-based superconducting slab with real E–J characteristic contend that the temperature of slab jumps from the coolant one to a peak one during flux jump and then back down the coolant one. This statement is not consistent with criteria (4) and (6).

Note that the stability analysis is the most favorable, if analytical expressions may be formulated. However, in practical cases, numerical methods are needed to find magnetic instability boundary since low- and high-temperature superconductors have essentially non-linear shape of E–J characteristics. Therefore, the numerical investigations of the macroscopic thermo-electrodynamics phenomena taking place in technical superconductors are used [24–26]. Nevertheless, such investigations are not numerous because they not only have very large computation time of the corresponding system of unsteady thermo-electrodynamics equations but require the development of appropriate methods of numerical calculations of such system of differential equations. In particular, it should be stressed that there is still no the modeling method, which allows one to make simulation of the unsteady electrodynamics states of type-II superconductor in the framework of the non-isothermal critical state model.