Physica C 500 (2014) 4–8

Contents lists available at ScienceDirect

Physica C

journal homepage: www.elsevier.com/locate/physc

Photonic band gap of superconductor-medium structure: Two-dimensional triangular lattice

Wan-guo Liu, Feng-ming Pan *, Li-wei Cai

Department of Applied Physics, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 211100, China

ARTICLE INFO

Article history:
Received 18 June 2013
Received in revised form 11 November 2013
Accepted 13 February 2014
Available online 26 February 2014

Keywords:
Photonic band gap
Superconductor
Plane wave expansion

ABSTRACT

Based on London theory a general form of wave equation is formulated for both dielectric medium and superconductor. Using the wave equation and applying plane wave expansion, we have numerically calculated the band structures and density of states of a photonic crystal, whose intersection is constructed by a two-dimensional triangular lattice of superconductor padding in dielectric medium. Results indicate a wider band gap in the superconductor-medium photonic crystal than that in conventional photonic crystals. And part of original energy levels are found to be rearranged upon consideration of the superconductivity. The dependence of band gap on penetration length and filling factor is also discussed. Band gap width decreases monotonically with the penetration length, but not with the filling factor. Band gaps can be partially shut down or opened by adjusting filling factor.

1. Introduction

Since the concept of photonic crystal was put forward by Yablonovitch [1] and John [2], there has been a growing interest in research of this spatially periodic dielectric medium in recent decades due to various modern applications [3–12]. Similar to the band gap of electron in periodic potential field, a frequency range in the photonic band structure, within which no photons exist, is so-called photonic band gap. A variety of theoretical approaches and numerical methods have been developed for solving the photonic band structure and transmission spectra of photonic crystals. The typical methods include plane wave expansion method [3–5], transfer matrix method [6–8], time-domain finite difference method [9] and KKR method [10–12]. Associated with absolute band gaps in this structure, photonic band gap materials have been found several potential applications in the field of band-pass filter [13,14], high reflection mirror [15], amplifier [16] and mixer [17]. The conventional photonic band gap materials are usually made up of dielectrics, metals, and semiconductors. The composite structure of these materials has a high transmission ratio and also possesses strong nonlinear optical effect so that it is widely used in photoelectric device. But the damping in this structure usually leads to high energy dissipation, making its potential application be restricted.

In recent years, superconducting materials are also introduced in the manufacture of photonic crystal. Their band structures and optical properties have been studied [18–20]. Compared with conventional photonic crystals, this new structure can lead to lower energy loss and ensure an ideal transmission ratio. Instead of adjusting the geometrical parameter of structure, the superconductive structure can get the same effect by controlling the temperature, which provides a great convenience in manufacture and application. Previous studies have investigated the transmission spectra and band structure of 1D photonic crystal with superconductor-medium structure by applying plane wave expansion method based on Bloch theorem [21–25], and superconductor-medium Fibonacci photonic crystal is also discussed by applying transfer matrix method [26]. In this paper we will show it is feasible to apply plane wave expansion method in 2D superconductor structure. The photonic band gap of 2D triangular lattice with superconductor padding is discussed in details. First we will deduce the wave equation in both dielectric medium and superconductor based on London theory. By solving the eigenvalue problem derived by plane wave expansion method, the band diagram and density of states are obtained. The difference between conventional photonic crystal and photonic crystal with superconductor padding in energy levels and band gaps will be presented.

* Corresponding author. Address: No. 29, Jiangjun Road, Jiangning District, Nanjing, Jiangsu, China. Tel.: +86 13851653956/13951911524.

E-mail address: fmpan@nuaa.edu.cn (F.-m. Pan).

http://dx.doi.org/10.1016/j.physc.2014.02.014
0921-4534/

2. Theoretical model of 2D triangular lattice with superconductor padding and numerical calculation by plane wave expansion method

The model of 2D triangular lattice with superconductor-medium structure is shown in Fig. 1. The superconductor can be viewed as the padding in dielectric medium. The position vector is written as $\vec{r} = x\vec{e}_x + y\vec{e}_y$, where $\vec{e}_x$ and $\vec{e}_y$ are unit vectors of x and y axes. The position vector of lattice node is given by $\vec{r}(\vec{l}) = l_1\vec{a}_1 + l_2\vec{a}_2$. The basis vectors of triangular lattice are expressed as $\vec{a}_1 = a(1,0); \vec{a}_2 = a(\frac{1}{2}, \frac{\sqrt{3}}{2})$, where a is lattice constant. The filling factor f is defined as $f = \frac{S_p}{S_c}$, where S_c is the area of Wigner–Seitz cell and S_p is the circular area of single superconductor padding. Obviously $f = \frac{2\pi R^2}{\sqrt{3}a^2}$, where R is filling radius of superconductor padding. The dielectric constant of dielectric medium and superconductor are ε_d and ε_s respectively.

When the London penetration length of superconductor $\lambda_L \gg \xi$, where ξ is coherent length, it is suitable to apply London approximation. In a lossless superconductor, the current density $\vec{j}(\vec{r})$ and electromagnetic vector potential $\vec{A}(\vec{r})$ obey the London equation

$$\vec{j}(\vec{r}) = -\frac{c}{4\pi\lambda_L}\vec{A}(\vec{r}) \tag{1}$$

For E_z polarization, substituting the current density given by (1) for the one in 2D Helmholtz equation, one can get the wave equation (in ideal dielectric medium there exists no current density).

$$\begin{aligned}&\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c^2}\varepsilon(\vec{r})\right]E(\vec{r},\omega)\\ &= j(\vec{r}) = \begin{cases} -\frac{1}{\lambda_L^2} i\omega A(\vec{r}) = -\frac{1}{\lambda_L^2}E(\vec{r},\omega) & \text{in super conductor}\\ 0 & \text{in medium}\end{cases}\end{aligned} \tag{2}$$

For simplicity the subscript z is omitted. Because the superconductor padding is circular, the wave equation in both superconductor and dielectric medium can be uniformly written as

$$-\frac{1}{\varepsilon(\vec{r})}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{\lambda_L^2}\sum_{\vec{l}} circ\left(\frac{\vec{r}-\vec{r}(\vec{l})}{R}\right)\right]E(\vec{r},\omega) = \frac{\omega^2}{c^2}E(\vec{r},\omega) \tag{3}$$

where $circ(\vec{r})$ is circle function. When $|\vec{r}| \leqslant 1$ it equals to 1 and $|\vec{r}| > 1$ it equals to 0. The summation goes over all of the lattice points on the whole plane xOy .

To solve Eq. (3), take Fourier expansion as follows

$$\frac{1}{\varepsilon(\vec{r})} = \sum_{\vec{G}} \kappa(\vec{G})e^{i\vec{G}\cdot\vec{r}} \tag{4}$$

$$\sum_{\vec{l}} circ\left(\frac{\vec{r}-\vec{r}(\vec{l})}{R}\right) = \sum_{\vec{G}} c(\vec{G})e^{i\vec{G}\cdot\vec{r}} \tag{5}$$

$$E(\vec{r},\omega) = \sum_{\vec{G}} A(\vec{k},\vec{G})e^{i(\vec{k}+\vec{G})\cdot\vec{r}} \tag{6}$$

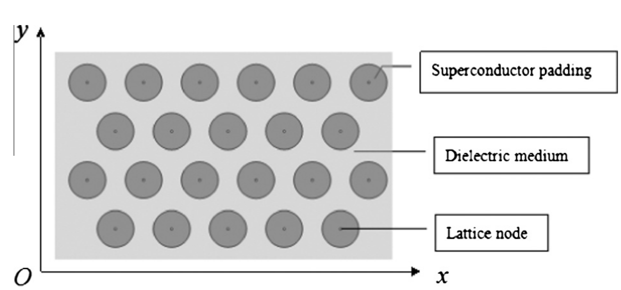

Fig. 1. The cross section of 2D triangular lattice with superconductor-medium structure.

where $\vec{k} = k_x\vec{e}_x + k_y\vec{e}_y$ is wave vector and $\vec{G} = h_1\vec{b}_1 + h_2\vec{b}_2$ is reciprocal lattice vector, $\vec{b}_1 = \frac{2\pi}{a}\left(1, -\frac{\sqrt{3}}{3}\right)$, $\vec{b}_2 = \frac{2\pi}{a}\left(0, \frac{2\sqrt{3}}{3}\right)$. One can plug (4)–(6) into (3). Noticed that in dielectric medium $\frac{1}{\varepsilon(\vec{r})}\sum_{\vec{l}} circ\left(\frac{\vec{r}-\vec{r}(\vec{l})}{R}\right) = 0$, and in superconductor padding $\varepsilon(\vec{r}) = \varepsilon_s$, this means $\frac{1}{\varepsilon(\vec{r})}\sum_{\vec{l}} circ(\frac{\vec{r}-\vec{r}(\vec{l})}{R}) = \frac{1}{\varepsilon_s}\sum_{\vec{G}} c(\vec{G})e^{i\vec{G}\cdot\vec{r}}$. Eq. (3) can be rewritten as

$$\begin{aligned}&\sum_{\vec{G}\vec{G}'}(\vec{k}+\vec{G}')^2 A(\vec{k},\vec{G}')\kappa(\vec{G})e^{i(\vec{k}+\vec{G}'+\vec{G})\cdot\vec{r}} - \frac{1}{\varepsilon_s\lambda_L^2}\sum_{\vec{G}\vec{G}'} c(\vec{G})A(\vec{k},\vec{G}')e^{i(\vec{k}+\vec{G}'+\vec{G})\cdot\vec{r}}\\ &= \frac{\omega^2}{c^2}\sum_{\vec{G}} A(\vec{k},\vec{G})e^{i(\vec{k}+\vec{G})\cdot\vec{r}}\end{aligned} \tag{7}$$

Here we use $\vec{G}$ and $\vec{G}'$ to distinguish the two summations of double sum. Because the summation goes over infinite plane, one can substitute $\vec{G} - \vec{G}'$ for $\vec{G}$ on the left side of Eq. (7), and compare the Fourier coefficient of $e^{i(\vec{k}+\vec{G})\cdot\vec{r}}$, the summation of $\vec{G}$ can be eliminated:

$$\sum_{\vec{G}'}[\kappa(\vec{G}-\vec{G}')(\vec{k}+\vec{G}')^2 - \frac{1}{\varepsilon_s\lambda_L^2}c(\vec{G}-\vec{G}')]A(\vec{k},\vec{G}') = \frac{\omega^2}{c^2}A(\vec{k},\vec{G}) \tag{8}$$

which has the form of the standard eigenvalue problem.

To solve Eq. (8) the Fourier coefficients c and κ should be calculated. For $c(\vec{G})$

$$\begin{aligned}c(\vec{G}) &= \sum_{\vec{l}}\frac{1}{S_c}\int_{S_c} circ\left(\frac{\vec{r}-\vec{r}(\vec{l})}{R}\right)e^{-i\vec{G}\cdot\vec{r}}dS\\ &= \frac{1}{S_c}\int_{xOy} circ\left(\frac{\vec{r}-\vec{r}(\vec{l})}{R}\right)e^{-i\vec{G}\cdot[\vec{r}-\vec{r}(\vec{l})]}dS = \frac{1}{S_c}\int_{S_p} e^{-i\vec{G}\cdot\vec{r}}dS\end{aligned} \tag{9}$$

One can apply the Sommerfeld integration form of Bessel function and rewrite (9) as

$$c(\vec{G}) = \begin{cases} f & |\vec{G}| = 0\\ 2f\frac{J_1(|\vec{G}|R)}{|\vec{G}|R} & |\vec{G}| \neq 0\end{cases} \tag{10}$$

where J_1 is Bessel function of order 1.

Similar to $c(\vec{G})$ by writing the dielectric constant $\varepsilon(\vec{r})$ as $\frac{1}{\varepsilon(\vec{r})} = \frac{1}{\varepsilon_d} + \left(\frac{1}{\varepsilon_s} - \frac{1}{\varepsilon_d}\right)\sum_{\vec{l}} circ(\frac{\vec{r}-\vec{r}(\vec{l})}{R})$, $\kappa(\vec{G})$ can be expressed as

$$\kappa(\vec{G}) = \begin{cases} \frac{1}{\varepsilon_s}f + \frac{1}{\varepsilon_d}(1-f) & |\vec{G}| = 0\\ \left(\frac{1}{\varepsilon_s} - \frac{1}{\varepsilon_d}\right)2f\frac{J_1(|\vec{G}|R)}{|\vec{G}|R} & |\vec{G}| \neq 0\end{cases} \tag{11}$$

With $c(\vec{G})$ and $\kappa(\vec{G})$ calculated, the eigenvalue problem described by Eq. (8) can be solved.

3. Results and discussion

Assume there exists no dispersion in dielectric medium and superconductor padding, $\varepsilon_s \approx 1$. First, let's review the band structure of conventional photonic crystal, which do not have superconductivity so London penetration length $\lambda_L = \infty$. The photonic band diagrams for the filling factor $f = 0.3$, the dielectric constant of dielectric medium $\varepsilon_d = 9$, is depicted in Fig. 2 $(X : \frac{\pi}{a}\left(1, \frac{\sqrt{3}}{3}\right)$, $\Gamma : (0,0), J : \frac{\pi}{a}\left(\frac{4}{3}, 0\right))$. From the band structure we also qualitatively obtain the density of state, which is proportional to the length of the curve segment extends between two crossings of the dispersion curve $\omega(k)$ with the boundaries of the small frequency segment $\Delta\omega$. It can be clearly seen that complete band gaps appears at the frequency $0.267 < \frac{\omega a}{2\pi c} < 0.378$, $0.500 < \frac{\omega a}{2\pi c} < 0.612$ and $0.766 < \frac{\omega a}{2\pi c} < 0.838$. The widths of first, second and third band gaps are $\omega_{g_1} = 0.111$, $\omega_{g_2} = 0.112$, $\omega_{g_3} = 0.072$, separately. The top and bottom of band gaps are all located at the boundary of irreducible Brillouin zone X, Γ and J, and for these positions the energy levels have degeneration.