Physica C 500 (2014) 14–19

Contents lists available at ScienceDirect

Physica C

journal homepage: www.elsevier.com/locate/physc

Flux-pinning-induced stresses in a hollow superconducting cylinder with flux creep and viscosity properties

W.J. Feng *, S.W. Gao

Department of Engineering Mechanics, Shijiazhuang Tiedao University, Shijiazhuang 050043, PR China

ARTICLE INFO

Article history:
Received 6 November 2013
Received in revised form 4 February 2014
Accepted 11 February 2014
Available online 19 February 2014

Keywords:
Stress
Viscosity
Flux creep
Hollow superconducting cylinder

ABSTRACT

The magnetoelastic problem for a superconducting cylinder with a concentric hole placed in a magnetic field is investigated, where the flux creep and viscous flux flow have been considered. The stress distributions are derived and numerical calculated for the descending field in both the zero-field cooling (ZFC) and field cooling (FC) processes. The effects of applied magnetic field, flux creep and viscous flux flow on the maximal radial and hoop stresses are discussed in detail, and some novel phenomena are found. Among others, for the FC case, the maximal hoop tensile stress always occurs at the hole edge, whist for the ZFC case, the maximal stresses including both hoop and radial stresses either occur in the vicinity of the hole or occur at the position of flux frontier in the remagnetization process. For the descending field, in general, both the flux creep and viscosity parameters have important effects on the maximal radial and hoop stresses. All these phenomena are perhaps of vital importance for the application of superconductors.

1. Introduction

In recent years, high-temperature superconductors (HTSs) have been developed for many applications [1–5]. Because flux-pinning-induced tensile stress exists in HTSs, the corresponding magneto-elastic problems of superconducting slabs, circular cylinders and/or circular disks have received considerable attention [6–15]. However, the critical state models adopted in those works only describe quasistatic flux distributions.

On the other hand, although either the thermally activated flux creep flow [16–18] or viscous flux [19–21] in superconductors has been, separately, considered, the reports on the effects of them together on the mechanical behaviors are very limited. To the best of our knowledge, only recently, Xue et al. [22] investigated the effects of both flux creep and viscous flux flow on the internal tensile stress and magnetostriction of a type-II superconducting slab. In addition, as well known, holes, as one kind of defects, have great effects on the stress distributions of superconductors [23,24], and tensile stress (which tends to generate cracks or expand already existing micro-cracks in the superconductors) always occurs in the process of field descent. Thus, based on the previous works [22,23], in this paper, we further investigate the magnetoelastic problem of an isotropic hollow superconducting cylinder subjected to the applied descending magnetic field, where the flux creep and viscosity of superconducting cylinder exists simultaneously. General expressions for stresses in terms of the flux density distribution are given firstly. The stress distributions are further derived for the descending field in both the zero-field cooling (ZFC) and field cooling (FC) activation processes. Numerous results are plotted and discussed in detail. The study should have help for the application of superconducting materials.

2. Basic equation

Consider a type-II superconducting cylinder of radius R with a concentric hole of radius a. The cylinder is placed in a time-dependent magnetic field B_a pointing to the z direction and assumed to be isotropic (see Fig. 1).

As known, some critical state models have been used to analyze the flux distribution in a superconductor. However, in a magnetization process, the flux velocity will increase with the magnetic field sweep rate [25], and the thermal activated flux creep will cause the critical state to relax away from its marginal stability because vortices in the superconductor usually jump out of their pinning centers [26]. In order to simultaneously take into account of the effects of flux creep and viscous flux flow on the superconductors in the process of field descent, the magnetic flux distribution in the considered hollow superconducting cylinder for the descending filed can be expressed as follows [22,25,26]:

* Corresponding author. Tel.: +86 31187936543.
E-mail address: wjfeng9999@126.com (W.J. Feng).

http://dx.doi.org/10.1016/j.physc.2014.02.013
0921-4534/© 2014 Elsevier B.V. All rights reserved.

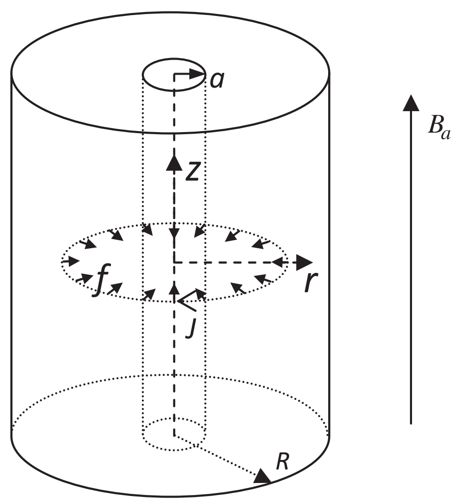

Fig. 1. A superconducting cylinder with a concentric hole placed in a parallel field B_a along the z-direction.

$$\frac{1}{\mu_0}\frac{\partial B}{\partial r} = J_c\left(\frac{|\dot{B}_a|}{\mu_0 J_c \upsilon_0}\right)^{1/(n+1)} + \frac{\eta}{\phi_0}\upsilon, \tag{1}$$

where B is the total magnetic field in the cylinder, $\dot{B}_a$ is the applied magnetic field sweep rate, J_c is the critical current density and equals a constant in the Bean model, η is the viscosity associated with flux motion, υ is the local flux flow velocity, ϕ_0 is the flux quantum, υ_0 is a constant relevant to flux creep velocity, n is a constant relevant to creep activation barrier, temperature and the Boltzmann constant, and μ_0 is the material constant as well. By the way, Eq. (1) implies that the slope of the flux distribution will increase with the velocity of flux flow, that the flux creep is linear, and that the creep activation barrier grows logarithmically with the decreasing current [22]. In addition, in Eq. (1), the demagnetization effect is simultaneously neglected because of the superconducting cylinder being infinitely long in the z direction.

According to the Maxwell equation, it is easily obtained

$$\frac{\partial B(r,t)}{\partial t} = -\frac{1}{r}\frac{\partial[r\upsilon(r,t)B(r,t)]}{\partial r}. \tag{2}$$

By integrating, one gets

$$r\upsilon(r,t)B(r,t) = -\int_0^r r'\frac{\partial B(r',t)}{\partial t}dr'. \tag{3}$$

Additionally, similar to the works before [19,22,25], by assuming that $\upsilon = \upsilon_1$ is a constant, we can further obtain from Eqs. (2) and (1) that $\frac{\partial B(r,t)}{\partial t} = -2\upsilon_1\frac{\partial B(r,t)}{\partial r}$, which means $B(r,t) = B(r - 2\upsilon_1 t)$ holds true. However, it should be pointed out that in the following section, for simplicity, although $B(r,t) = B(r - 2\upsilon_1 t)$ is used, strictly speaking, the relation does not satisfy Eq. (3). In fact, $B(r,t) = B(r - 2\upsilon_1 t)$ essentially assumes that the flux density linearly relies on both r and t, which can be easily seen latter (See Eqs. (12) and (21)).

The stress components including radial stress σ_r and hoop stress σ_θ in an infinitely long hollow cylinder placed in a parallel magnetic field have been investigated by Johansen el al. [23]. And they are, respectively,

$$2\mu_0\sigma_r = B^2 - B_h^2 + \frac{1-(a/r)^2}{1-(a/R)^2}(B_h^2 - B_a^2) + \frac{1-2v}{1-v}\left[\frac{1-(a/r)^2}{R^2-a^2}\int_a^R r'B^2dr' - \frac{1}{r^2}\int_a^r r'B^2dr\right], \tag{4}$$

$$2\mu_0\sigma_\theta = \frac{v}{1-v}B^2 - B_h^2 + \frac{1+(a/r)^2}{1-(a/R)^2}(B_h^2 - B_a^2) + \frac{1-2v}{1-v}\left[\frac{1+(a/r)^2}{R^2-a^2}\int_a^R r'B^2dr' + \frac{1}{r^2}\int_a^r r'B^2dr'\right], \tag{5}$$

where B_h is the magnetic field at the position $r = a$, E and v are the Young's modulus and Poisson's ratio of the superconducting cylinder, respectively.

3. Stress distributions

3.1. ZFC

For the ZFC process, it is assumed that the applied field is reduced from its maximal value $B_m = 4B_p$ ($B_p = \mu_0 J_c R$ is an introduced characteristic field corresponding to the full penetration field of a solid cylinder without the defect in the Bean model) to B_a. Here there are two situations decided by B_a should be considered. (i) The negative and positive currents are existent simultaneously in the superconducting cylinder with a concentric hole. (ii) The currents are all positive in the superconducting cylinder. Set B^* be the parameter field when the currents change from (i) to (ii). The dimensionless b^* is defined and easily obtained as

$$b^* = \frac{B^*}{B_p} = b_m - (1-\bar{a})(\chi+1), \tag{6}$$

where the dimensionless quantities χ, b_m, and $\bar{a}$ are, respectively, defined as

$$\chi = \left(|\dot{b}_a|\frac{R}{\upsilon_0}\right)^{1/(n+1)} + \frac{\eta\upsilon_1}{\phi_0 J_c}, \tag{7}$$

$$b_m = B_m/B_p, \quad \bar{a} = a/R \tag{8}$$

with $\dot{b}_a = \dot{B}_a/B_p$.

After further introducing the following dimensionless quantities

$$b = B/B_p, \quad b_a = B_a/B_p, \quad b_h = B_h/B_p, \tag{9}$$

$$\rho = r/R, \quad \rho_0 = r_0/R, \tag{10}$$

substituting $B(r,t) = B(r - 2\upsilon_1 t)$ into Eq. (1), and using boundary condition

$$b(1,t) = b_a(t), \quad b_a(0) = b_m, \tag{11}$$

the flux densities for the two cases can be, respectively, written as follows:

Case (i): $b^* < b_a(t) \leqslant b_m$

$$b(\rho,t) = \begin{cases} b_m + \rho - 1, & \bar{a} \leqslant \rho \leqslant \rho_0, \\ b_a(t) + \chi(1-\rho), & \rho_0 \leqslant \rho \leqslant 1, \end{cases} \tag{12}$$

where

$$b_a(t) = b_m - 2\chi\frac{\upsilon_1}{R}t, \tag{13}$$

$$\rho_0 = 1 - (b_m - b_a(t))/(\chi+1); \tag{14}$$

Case (ii): $0 \leqslant b_a(t) \leqslant b^*$

$$b(\rho,t) = b_a(t) + \chi(1-\rho), \bar{a} \leqslant \rho \leqslant 1. \tag{15}$$

Substituting Eqs. (12) and (15) into Eqs. (4) and (5), we can obtain the stress distributions easily. For example, as $b^* \leqslant b_a \leqslant b_m$,