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Multiqubit quantum phase gate using four-level superconducting quantum interference devices coupled to superconducting resonator

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ABSTRACT

In this paper, we propose a scheme to realize three-qubit quantum phase gate of one qubit simultaneously controlling two target qubits using four-level superconducting quantum interference devices (SQUIDs) coupled to a superconducting resonator. The two lowest levels $|0\rangle$ and $|1\rangle$ of each SQUID are used to represent logical states while the higher energy levels $|2\rangle$ and $|3\rangle$ are utilized for gate realization. Our scheme does not require adiabatic passage, second order detuning, and the adjustment of the level spacing during gate operation which reduce the gate time significantly. The scheme is generalized for an arbitrary n-qubit quantum phase gate. We also apply the scheme to implement three-qubit quantum Fourier transform.

1. Introduction

Quantum information processing has the potential ability to simulate hard computational problems much more efficiently than classical computers. For example factorization of large integers [1], searching for an item from disordered data base [2], and phase estimation [3]. Quantum logical gates based on unitary transformations are building blocks of quantum computer. The schemes for realizing two-qubit quantum logical gates using physical qubits such as atoms or ions in cavity QED [4,5], superconducting devices like Josephson junctions [6], cooper pair boxes [7] have been proposed. In earlier studies, strong coupling with charge qubits [8] and flux qubits [9] was predicted in circuit QED. In some recent studies, experimental demonstration of strong coupling in microwave cavity QED with superconducting qubit [10–12] has been realized. The results of these experiments make superconducting qubit cavity QED an attractive approach for quantum information processing.

Among the superconducting state qubits, SQUIDs are promising candidate to serve as a qubit [13]. They have long decoherence time of the order of 1–5 μs [14,15], design flexibility, large-scale integration, and compatibility to conventional electronics [14,16,17]. They can be easily embedded in the cavity while atoms or ions require trapping techniques. Some interesting schemes for the realization of two-qubit quantum controlled phase gates based on a cavity QED technique with SQUIDs have been proposed [18–21]. These studies open a way of realizing physical quantum information processing with SQUIDs in cavity QED.

Recently, physical realization of the multiqubit gates has gained a lot of interest [22–25]. Algorithms for quantum computing become complex for large qubit system. However, multiqubit quantum phase gate reduces their complexity and can lead to faster computing. Multiqubit quantum controlled phase gate has great importance for realizing quantum-error-correction protocols [26], constructing quantum computational networks [27] and implementing quantum algorithms [28].

In this paper, we present a scheme for the realization of three-qubit quantum controlled phase gate of one-qubit simultaneously controlling two qubits using four-level SQUIDs coupled to a superconducting resonator. It may be mentioned that in an earlier study, a proposal for multiqubit phase gate of one qubit simultaneously controlling n qubits in a cavity has been presented [24], which is based upon system-cavity-pulse resonant Raman coupling, system-cavity-pulse off-resonant Raman coupling, system-cavity off-resonant interaction and system-cavity resonant interaction. In another study [25], a multiqubit phase gate based upon the tuning of the qubit frequency or resonator frequency is proposed. These proposals are quite general which can be applied to flux qubit systems or SQUIDs too. The present scheme is based on system-cavity-pulse resonant and system-cavity off-resonant interactions which can be realized using flux qubit (SQUID) system. In this proposal, two lowest levels $|0\rangle$ and $|1\rangle$ of each SQUID are used to represent the logical states while higher energy levels $|2\rangle$ and $|3\rangle$ are utilized for gate realization. A single photon is created by

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resonant interaction of cavity field with $|2\rangle \leftrightarrow |3\rangle$ transition of the control SQUID. In the presence of single photon inside the cavity, off-resonant interaction between the cavity field and $|2\rangle \leftrightarrow |3\rangle$ transition of each target SQUID induces a phase shift of $e^{i\theta_n}$ to each nth target SQUID. Our scheme has following advantages:

(1) Controlled phase gate operation can be performed without adjusting level spacing during gate operation, thus decoherence due to tuning of SQUID level spacing is avoided.
(2) The proposal does not require slowly changing Rabi frequencies (to satisfy adiabatic passage) and use of second-order detuning (to achieve off-resonance Raman coupling between two relevant levels), thus the gate is significantly faster.
(3) During the gate operation, tunneling between the levels $|1\rangle$ and $|0\rangle$ is not needed. The decay of level $|1\rangle$ can be made negligibly small via prior adjustment of the potential barrier between the levels $|1\rangle$ and $|0\rangle$ [29]. Therefore, each qubit can have much longer storage time.
(4) We do not require identical coupling constants of each SQUID with the resonator. Similarly, detuning of the cavity modes with the transition of the relevant levels in every target SQUID is not identical, therefore, our scheme is tolerable to inevitable non-uniformity in device parameters.

The scheme is generalized to realize n-qubit quantum controlled phase gate. Finally, it is shown that the proposed scheme can be used to implement three-qubit quantum Fourier transform (QFT).

2. Quantum phase gate

The transformation for three-qubit quantum phase gate with one qubit simultaneously controlling two target qubits is given by:

$$U_3|q_1,q_2,q_3\rangle = e^{(i\theta_2\delta_{q_1,1}\delta_{q_2,1})}e^{(i\theta_3\delta_{q_1,1}\delta_{q_3,1})}|q_1,q_2,q_3\rangle, \tag{1}$$

where $|q_1\rangle$, $|q_2\rangle$, and $|q_3\rangle$ stand for basis states $|1\rangle$ or $|0\rangle$ for qubits 1, 2, and 3, respectively. Here $\delta_{q_1,1}$, $\delta_{q_2,1}$, and $\delta_{q_3,1}$ are the Kronecker delta functions. It is clear from Eq. (1) that in three-qubit quantum phase gate when control qubit $|q_1\rangle$ is in state $|1\rangle$, phase shift $e^{i\theta_2}$ induces to the state $|1\rangle$ of the target qubit $|q_2\rangle$ and phase shift $e^{i\theta_3}$ to the state $|1\rangle$ of target qubit $|q_3\rangle$. When control qubit $|q_1\rangle$ is in state $|0\rangle$ nothing happens to the target qubits. Quantum phase gate operator in Dirac notation can be written as:

$$\begin{aligned} U_3 = {} & |000\rangle\langle 000| + |001\rangle\langle 001| + |010\rangle\langle 010| + |011\rangle\langle 011| \\ & + |100\rangle\langle 100| + e^{i\theta_3}|101\rangle\langle 101| + e^{i\theta_2}|110\rangle\langle 110| \\ & + e^{i\theta_2}e^{i\theta_3}|111\rangle\langle 111|. \end{aligned} \tag{2}$$

The schematic circuit diagram for quantum phase gate with one qubit simultaneously controlling two target qubits is shown by circuit-1 in Fig. 1. The circuit-2 in Fig. 1 shows the two successive two-qubit controlled phase gate represented by U_2 and U_3 with shared target qubit (i.e., qubit $|q_1\rangle$) but different control qubits $|q_2\rangle$ and $|q_3\rangle$ (as shown by filled circles). The circuit-2 is known as gate decomposition method. The elements U_2 and U_3 represent controlled phase gate having phase shift $e^{i\theta_2}$ and $e^{i\theta_3}$, respectively. These circuits are equivalent to each other [24] which can provide fast implementation of QFT as discussed in Section 6.

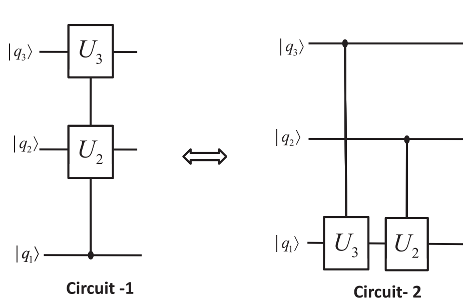

Fig. 1. Circuit-1 shows quantum phase gate with one-qubit $|q_1\rangle$ simultaneously controlling two target qubits $|q_2\rangle$ and $|q_3\rangle$. Circuit-2 shows the two successive two-qubit controlled phase gate with shared target qubit $|q_1\rangle$. The elements U_2 and U_3 represent two qubit controlled phase gate of phase shift $e^{i\theta_2}$ and $e^{i\theta_3}$, respectively. These circuits are equivalent to each other.

3. Dynamics of the system

Here we consider rf-SQUIDs which consists of Josephson junction enclosed by superconducting loop. The corresponding Hamiltonian is given by [30]:

$$H_S = \frac{Q^2}{2C} + \frac{(\phi - \phi_x)^2}{2L} - E_J\cos\left(\frac{2\pi\phi}{\phi_0}\right), \tag{3}$$

where C and L are junction capacitance and loop inductance, respectively. Conjugate variables of the system are magnetic flux ϕ threading the ring and total charge Q on capacitor. The static external flux applied to the ring is ϕ_x and $E_J \equiv \frac{I_c\phi_0}{2\pi}$ is the Josephson coupling energy. Here I_c is critical current of Josephson junction and $\phi_0 = \frac{h}{2e}$ is the flux quantum. We consider the interaction of SQUID with cavity field and microwave pulses as discussed in the forthcoming subsections.

3.1. Control SQUID interaction with resonator

Control SQUID is biased properly to achieve desired four-level structure by varying the external flux [21] as shown in Fig. 2. The single-mode of the cavity field is resonant with $|2\rangle_1 \rightarrow |3\rangle_1$ transition of control SQUID, however, it is highly detuned from the transition between the other levels which can be achieved by adjusting the level spacing of SQUID [18,29]. Using interaction picture with rotating wave approximation one can write the Hamiltonian of system as [18]:

$$H_1 = \hbar(g_1 a^\dagger |2\rangle_1\langle 3| + H.c), \tag{4}$$

where $a^\dagger$ and a are photon creation and annihilation operators for the cavity field mode of frequency ω_c. Here g_1 is the coupling constant between cavity field and $|2\rangle_1 \rightarrow |3\rangle_1$ transition of the control SQUID. The evaluation of initial states $|3\rangle_1|0\rangle_c$ and $|2\rangle_1|1\rangle_c$ under Eq. (4) can be written as:

$$\begin{aligned} &|3\rangle_1|0\rangle_c \rightarrow cos(g_1t)|3\rangle_1|0\rangle_c - isin(g_1t)|2\rangle_1|1\rangle_c, \\ &|2\rangle_1|1\rangle_c \rightarrow cos(g_1t)|2\rangle_1|1\rangle_c - isin(g_1t)|3\rangle_1|0\rangle_c, \end{aligned} \tag{5}$$

where $|0\rangle_c$ and $|1\rangle_c$ are vacuum and single photon states of the cavity field, respectively.

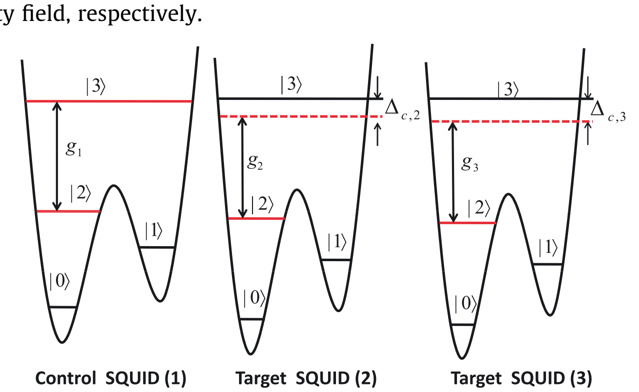

Fig. 2. Level diagram of control SQUID and target SQUIDs with four levels $|0\rangle$, $|1\rangle$, $|2\rangle$, and $|3\rangle$. The levels $|2\rangle$, and $|3\rangle$ of control SQUID interact resonantly to resonator while levels $|2\rangle$ and $|3\rangle$ of each target SQUID interact off-resonantly to the resonator. The difference between level spacing of each SQUID can be achieved by choosing different device parameters for SQUIDs.