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Domains in multiband superconductors

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ABSTRACT

Multiband superconductors can have several types of domains that are inhibited in conventional single-band superconductors. These domains are phase domains and chiral domains and their domain wall are an interband phase difference soliton. In a superconductor with an odd number of electronic bands (five or more) and with positive interband Josephson interactions, we find other types of domains with different interband phase differences. We call these domains configuration domains because pseudo-order parameters for each band are dispersed in the complex plain and several configurations, which have several local minima. Fractional vortices serve as hubs for phase difference solitons (configuration domain walls). The divergence of the number of configurations with local minima would pose a serious problem for the stability of superconductivity.

1. Introduction

Conventional superconductors exhibit one-dimensional unitary ($U(1)$) internal space, which is a superconducting quantum phase θ [1–3]. When the interband Josephson interaction is much smaller than intraband interactions, multiband superconductors can be considered to be multicomponent superconductors that have multiple quantum phases θ_ν, where ν is the band index [4–8]. These can be called multicomponent superconductors based on the multiband superconductor in which the superconducting order parameters are composed of pseudo-order parameters $\psi_\nu = |\psi_\nu| \exp(i\theta_\nu)$ for each band, where $|\psi_\nu|^2$ is the pair density. By applying the extended London approximation to this system [7–9], we had presented characteristic phenomena such as an interband phase difference soliton and fractional flux quanta [7–13]. In the extended London approximation, the amplitude of the pseudo-order parameters $|\psi_\nu|^2$ is held constant. This approximation is based on the fact that the crystal fields (or a band structure) determine the number of carriers for each band.

Sometimes, multicomponent superconductivity based on multiband superconductors is considered as a symmetry-breaking state of a $U(1)^N$ system, where N is the number of bands [14–17]. However, the interband interaction *drags* other gauge fields on higher-dimensional manifolds (such as a manifold having broken $U(N)$ symmetry) [18–21]. For example, in a two-band superconductor, we can identify the electromagnetic $U(1)$ gauge field and three other gauge fields. Because of the extended London approximation and the real current condition $J = 0$, two degrees of freedom are removed; and the Josephson interaction combines three gauges into a single gauge field [21]. This is the nonmagnetic gauge field for the interband phase difference, and sometimes (depending on the ratio of $m_\nu^*/|\psi_\nu|^2$ between two bands, where m_ν^* is the mass of the pair) the manifold for this gauge is noncompact. In three-band superconductors, the potential surface determined by the Ginzburg–Landau free energy reduces the number of gauge fields [13]. The omitted gauge fields determine the dynamics of the soliton, such as its vibration and deformation in the quantum phase space, which are ignored in the discussion of the static and semistatic situation (e.g., a slow translational movement of the soliton). In multiband superconductors, we consider that incrementing the number of internal degrees of freedom, the constraints from crystal fields, and the potential surface of the Ginzburg–Landau free energy primarily determines topology of the manifold. The high symmetry and its breakdown do not have complete informations [22–29].

Recently, experimental observations that indicate the existence of these new internal degrees of freedom have been reported

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[30–35]. For magnetization, phase difference solitons and fractional vortices have been clearly demonstrated by scanning microscopy [30,32,33]. In macroscopic experiments, a curiously strong pinning effect was found in some new superconductors, which raises the suspicion of domain trapping of fractional vortices at the domain wall [12,36]. There are several mechanisms that lead to chiral superconductivity in which we find chiral domains [12,22,24,25,27,29,37]. The chiral states suggested by three-band superconductivity are prominent [13,14,16,38–41] (we were aware of reports [39,40] on these when we published our reports on chiral superconductivity based on three-band superconductors [13,41]). In particular, in a recent report [40], neither the high crystal symmetry nor a multidimensional representation due to this high symmetry is provided. Nondegenerate multiple bands lead to chiral states by the positive interband Josephson interaction. From this fact, we can confirm that the multiplicity itself is primary rather than the high symmetry and its breakdown. Based on this understanding, we demonstrate a new type of domain, which we refer to hereinafter as the configuration domain of multiband superconductors.

2. Model formula

The extended Ginzburg–Landau formalism is convenient for dealing with multiband superconductors. We also assume the extended London approximation. With this simplification, we can understand the physics on a more intuitive level [9,12,13].

The phenomenological free energy f can be expressed as

$$f=\left(\sum_{\nu=1}^{N}\left(\alpha_\nu|\psi_\nu|^2+\frac{\beta_\nu}{2}|\psi_\nu|^4+\frac{1}{2m_{\nu*}}\left|\left(\frac{\hbar}{i}\nabla-\frac{e^*}{c}A\right)\psi_\nu\right|^2\right)\right)+\sum_{\mu=1}^{N}\left(\sum_{\nu=1,\nu\neq\mu}^{N}\frac{\gamma_{\mu\nu}}{4}\left(\psi_\mu^+\psi_\nu+\psi_\nu^+\psi_\mu\right)\right)+\frac{h^2}{8\pi}, \quad (1)$$

where N is the number of bands, e^* is the charge of the pair, $m_{\nu}*$ is mass, A is the vector potential, and c is the speed of light. The sum $\sum_{\nu=1}^{N}\alpha_\nu|\psi_\nu|^2+\frac{\beta_\nu}{2}|\psi_\nu|^4$ gives the energy due to intraband interactions, and $\frac{h^2}{8\pi}$ is the energy of the magnetic field. The kinetic energy term, $\sum_{\nu=1}^{N}\frac{1}{2m_\nu^*}\left|\left(\frac{\hbar}{i}\nabla-\frac{e^*}{c}A\right)\psi_\nu\right|^2$, can be removed to explore the homogeneous state. The interband Josephson interactions $\frac{\gamma_{\mu\nu}}{4}(\psi_\mu^+\psi_\nu+\psi_\nu^+\psi_\mu)$ determine the relative phase in the homogenous ground state, where μ and ν are the band indices [12,13].

To discuss the ground state, we can also neglect the magnetic contribution. With the extended London approximation, f can be considerably simplified, as follows [12,13]:

$$f=f_0+f_\theta, \quad (2)$$

$$f_0=\sum_{\nu=1}^{N}\left(\alpha_\nu|\psi_\nu|^2+\frac{\beta_\nu}{2}|\psi_\nu|^4\right), \quad (3)$$

$$f_\theta=\sum_{\mu=1}^{N}\left(\sum_{\nu=1,\nu\neq\mu}^{N}\frac{\gamma_{\mu\nu}}{2}|\psi_\mu||\psi_\nu|\cos(\theta_\nu-\theta_\mu)\right)=\sum_{\mu=1}^{N}\left(\sum_{\nu=1,\nu\neq\mu}^{N}\frac{\Gamma_{\mu\nu}}{2}\cos(\theta_\nu-\theta_\mu)\right). \quad (4)$$

When the interband interaction is much smaller than the intraband interaction, we can consider $|\psi_\nu|$ is determined by f_0. Introducing $\Gamma_{\mu\nu}=|\psi_\mu||\psi_\nu|\gamma_{\mu\nu}$, we can discuss the quantum phase using f_θ. As seen in previous reports, the positive interband Josephson interaction results in the nontrivial wave leading to chiral superconductivity. We also explore the same situation for multiband superconductors having four or more bands. For simplicity, we assume the multiple equivalent band, and let $\Gamma_{\mu\nu}=\Gamma>0$ for all μ and ν. In this discussion, we carefully inspect the effect of high symmetry due to this simplification and distinguish between the effect due to symmetry and that due to multiplicity.

3. Results

Eq. (4) has local maxima or minima for some values that satisfy $\partial f_\theta/\partial\theta_\nu=0$. The condition $\theta_\nu-\theta_\mu=0$ or π gives trivial solutions for f_θ. When all the interband phase differences are zero, the trivial solution corresponds to the s_{++} wave. In contrast, when some of the phase differences are π, the trivial solution is an $s_\pm$ wave. Because we assume a positive interband Josephson interaction, the s_{++} wave gives the highest energy. Among several $s_\pm$ waves, the one that corresponds to half the term in the sum of Eq. (4) having $\theta_\mu=0$ and half having $\theta_\mu=\pi$ when N is even. When N is three or more and is odd, this minimum energy is given by $\theta_\mu=0$ for $(N+1)/2$ bands with the other bands having $\theta_\mu=\pi$. We designate this wave $s_{0\pm}$. When there are an odd number of bands, we have a local minimum at $|\theta_{\mu+1}-\theta_\mu|=\frac{2\pi}{N}$ for every μ, where $\theta_{N+1}=\theta_1$. This wave is designated s_* and is shown as an example in Fig. 1a and b for $N=3$ and 5, respectively.

We also observe $f_\theta(s_{0\pm})-f_\theta(s_*)=\frac{\Gamma}{2}$, which means that the wave s_* is more stable than the wave $s_{0\pm}$. When there are an even number of bands, $f_\theta(s_{0\pm})-f_\theta(s_*)=0$. We show the wave s_* for $N=4$ in Fig. 1c. Moreover, several other waves have the same energy as $f_\theta(s_{0\pm})$. For $N=4$, $\theta_1-\theta_3=\theta_2-\theta_4=\pi$ and $\theta_1-\theta_2$ can be arbitrary, as shown in Fig. 1d. This can be designated $s_\pm+e^{i\theta}s'_\pm$. Eq. (4) cannot lock $\theta_1-\theta_2$. Rotation of $\theta_1-\theta_2$ corresponds to the Nambu–Goldstone mode [42]. To lock in this value, some $\Gamma_{\mu\nu}$ values should differ from other $\Gamma_{\mu\nu}$ values (for example, $\Gamma_{13}=\Gamma_{14}=\Gamma_{23}=\Gamma_{24}>\Gamma_{12}=\Gamma_{34}$ or $\Gamma_{12}=\Gamma_{24}=\Gamma_{13}=\Gamma_{34}>\Gamma_{23}=\Gamma_{14}$). In this case, $\theta_1-\theta_2$ will be locked at 0 or π and $f_\theta(s_{0\pm})$ becomes stable. In the locked phase, the Nambu–Goldstone mode becomes a phase difference soliton. This instability also occurs for other larger even numbers of bands. To lock in the interband phase difference, some different $\Gamma_{\mu\nu}$ are required (for example, $\Gamma_{\mu\leqslant\frac{N}{2}\nu>\frac{N}{2}}>\Gamma_{\mu\leqslant\frac{N}{2}\nu\leqslant\frac{N}{2}}=\Gamma_{\mu>\frac{N}{2}\nu>\frac{N}{2}}$) for higher even numbers of bands.

4. Discussion

It has been argued that the pure repulsive pair-exchange interaction mediates superconductivity in multiband superconductors [43–46]. The present model suggests the repulsive channel stabilizes $s_\pm$ for an even number of bands without any special symmetry. For some high-symmetry systems, $s_\pm+e^{i\theta}s'_\pm$ might be possible, and such a system could be interesting if it were considered to be real superconductivity. However, we currently do not have any realistic candidate for such a system, and it may be rather difficult to realize this condition (or it might correspond to the "non-superconducting/incoherent" pseudogap state in the cuprate superconductors.[47,48]). Thus, this possibility is not interesting for investigations of the present *coherent* superconductivity.

However, an s_* wave for an odd-numbered system should be considered if we are to understand the real phenomenon because the minimum at $|\theta_{\mu+1}-\theta_\mu|=\frac{2\pi}{N}$ does not disappear even if some $\Gamma_{\mu\nu}$ values change slightly. The system should retain many local minima of which $(N-1)!$ diverge upon increasing the number of bands. When we have only three bands, these local minima are recognized as different chiral states; but when the number exceeds three, many other configurations that are different from the chiral state become possible. For example, we show varying configurations in Fig. 2a–d for $N=5$. The energy of these four configurations gives the local minimum. Even if all $\Gamma_{\mu\nu}$ values are not equal, the local minimum does not disappear when all $\Gamma_{\mu\nu}$ values are similar.