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Full-wave modeling of superconducting microstrip lines including the nonlinearity behavior

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ABSTRACT

This paper describes a new theoretical model to characterize the superconducting microstrip line and carefully studies the effects of the nonlinearity of superconductors, the strip thickness and losses on circuit performances. The microstrip line has been considered as a multilayered structure. The integral equation for the electrical field has been formulated, in the spectral domain, using the exact dyadic Green's function of bianisotropic planar media. The Galerkin's technique has been used for solving this integral equation. Obtained results concern the effective permittivity constant and the attenuation constant versus frequency and temperature rate.

1. Introduction

The interest in applications of microwave components and devices based on HTS films initiated a new wave of investigations of physical phenomena in the superconducting films. In many cases, nonlinear effects limit high-power handling of the HTS films in linear microwave devices and therefore are desired to be suppressed. However, the nonlinear phenomena can be considered as useful effects in the design of microwave signal limiters or frequency mixers [1–5].

The high-temperature superconducting microstrip lines have been characterized by using the simplified quasi-static approach. Lee and Itoh [2] have proposed a method (phenomenological loss equivalence method) to analyze the structures whose strip thickness was in the order of the penetration depth. Another approach, developed by Nghiem et al. [6], considers the superconducting strip as equivalent surface impedance.

Almost all the developed approaches consider the effect of small losses in, or the anisotropy of, the superconducting strip material as perturbational effects that may not substantially alter the performance of these devices [3–18].

The analysis of nonlinearity in the superconducting microstrip line on lossy dielectric substrates requires an accurate full-wave model accounting for nonlinearity, electromagnetic coupling, dielectrics and conductors losses.

In this paper, we have developed a new theoretical model to study the nonlinearity in superconducting microstrip line on lossy substrate and carefully study the effects of the dielectric structure.

New integral equations for the electrical field components are formulated, in the spectral domain, using the exact dyadic Green's function of a bianisotropic planar media, applied to the superconducting microstrip lines. The assumption that the superconductor is not linear necessitates the addition of a second term in the second member of the electrical field integral equation. This condition to the limit is imposed along the width of the strip (W), by the integration on W. Thereby, we obtain a new characteristic equation consequent of the tangential electrical field components. The characteristic equation is formulated, in spectral domain, using the developed exact dyadic Green's function. This equation is solved using the two dimensional Galerkin's technique. The characteristics of the superconducting microstrip are obtained by the cancellation of the determinant of the resulting homogenous matrix equation.

2. Theoretical background

Consider the superconducting microstrip line shown in Fig. 1. The substrate, considered as an arbitrary lossy layer, extends to infinity in x- and y-directions. The thickness of the metallization is considered. Then, the surface current is assumed to flow only in x-, and y-directions in the strip.

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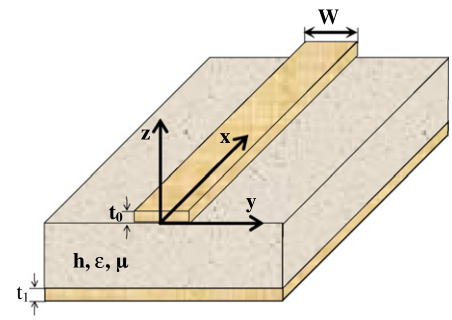

Fig. 1. Superconductor microstrip line.

Using the Maxwell's formulation and applying the three-dimensional dyadic Green's function of planar bianisotropic media, to the lossy microstrip, yield the electrical field integral equation given by:

$$\vec{E}(y,z) = \int_{z_0}\int_{y_0} \overline{\overline{G}}(y,z,k_s/y_0,z_0)\cdot \vec{J}_s(y_0,z_0)dy_0dz_0 \tag{1}$$

where $\vec{J}_s(y_0,z_0)$ is the surface current, and $\overline{\overline{G}}(y,z/y_0,z_0)$ is the two-dimensional dyadic Green's function, expressed as follows [5,14]:

$$\overline{\overline{G}}(y,z,k_s/y_0,z_0) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \times \widehat{g}(k_y,k_z,k_s)e^{jk_y(y-y_0)}e^{jk_z(z-z_0)}dk_ydk_z \tag{2}$$

We note here that our assumption of the superconducting microstrip line as a bianisotropic planar medium requires, to characterize it, an infinitely long bianisotropic microstrip line of width W, with a longitudinal distribution current of the form:

$$f(x) = e^{jk_sx} \tag{3}$$

The analytical integration of electrical field integral equation with respect to x, gives the complex propagation constant k_s. The adoption of the dyadic Green's function of a bianisotropic medium, for characterizing the microstrip structures of lossy multilayered substrate and superstrate, and with laminated ground plane, necessitates the development of new permittivity and permeability functions in space domain as:

$$\begin{aligned}\varepsilon(x,y,z) = {} & \varepsilon_0 - \sum_{k=0}^{m} j\frac{\sigma_k}{\omega}\cdot P_{W/2}[y\cdot\delta(k)] \\ & \cdot P_{t_k/2}\left[z-\frac{t_k}{2}+(1-\delta(k))\cdot\left(t_k+\sum_{i\leqslant N}h_i\right)+\sum_{0<i<k}t_i\right] \\ & +\sum_{i\leqslant N}\varepsilon_0\varepsilon_{r_i}\left(1-\frac{1}{\varepsilon_{r_i}}-j\tan\delta_i\right)\cdot P_{h_i/2}\left[z+\frac{h_i}{2}+\sum_{j<i}h_j\right] \\ & +\sum_{i\leqslant N_s}\varepsilon_0\varepsilon_{rs_i}\left(1-\frac{1}{\varepsilon_{rs_i}}-j\tan\delta_{s_i}\right) \\ & \cdot\left[P_{h_{s_i}/2}\left(z-\frac{h_{s_i}}{2}-\sum_{j<i}h_{s_j}\right)-P_{W/2}(y)\cdot P_{t_0/2}\left(z-\frac{t_0}{2}\right)\right]\end{aligned} \tag{4}$$

$$\begin{aligned}\mu(x,y,z) = {} & \mu_0+\sum_{k=0}^{m}\mu_0\mu_{rc_k}\left(1-\frac{1}{\mu_{rc_k}}-j\tan\delta_{MC_k}\right)\cdot P_{W/2}[y\cdot\delta(k)] \\ & \cdot P_{t_k/2}\left[z-\frac{t_k}{2}+(1-\delta(k))\cdot(t_k+\sum_{i\leqslant N}h_i)+\sum_{0<i<k}t_i\right] \\ & +\sum_{i\leqslant N}\mu_0\mu_{r_i}\left(1-\frac{1}{\mu_{r_i}}-j\tan\delta_{M_i}\right)\cdot P_{h_i/2}\left[z+\frac{h_i}{2}+\sum_{j<i}h_j\right] \\ & +\sum_{i\leqslant N_s}\mu_0\mu_{rs_i}\left(1-\frac{1}{\mu_{rs_i}}-j\tan\delta_{Ms_i}\right) \\ & \cdot\left[P_{h_{s_i}/2}\left(z-\frac{h_{s_i}}{2}-\sum_{j<i}h_{s_j}\right)-P_{W/2}(y)\cdot P_{t_0/2}\left(z-\frac{t_0}{2}\right)\right]\end{aligned} \tag{5}$$

where N is the number of substrate layers characterized by their thicknesses h_i, permeabilities $\mu_0\mu_{r_i}(1-j\tan\delta_{M_i})$ and permittivities $\varepsilon_0\varepsilon_{r_i}(1-j\tan\delta_i)$, N_s is the number of superstrate layers characterized by their thicknesses h_{si}, permeabilities $\mu_0\mu_{rs_i}(1-j\tan\delta_{Ms_i})$ and permittivities $\varepsilon_0\varepsilon_{rs_i}(1-j\tan\delta_{s_i})$ and **m** is the number of lamina composite ground plane characterized by their thicknesses t_k, permeabilities $\mu_0\mu_{rc_k}(1-j\tan\delta_{MC_k})$ and permittivities $(\varepsilon_0-j\frac{\sigma_k}{\omega})$. We define the functions $P_\tau(x)$ and $\delta(x)$ as follows:

$$P_\tau(x) = \begin{cases}1 & \text{for } -\tau/2\leqslant x\leqslant\tau/2\\ 0 & \text{otherwise}\end{cases} \tag{6}$$

$$\delta(x) = \begin{cases}1 & \text{for } x=0\\ 0 & \text{otherwise}\end{cases} \tag{7}$$

We can apply the developed permittivity and permeability functions expressed by (4) and (5) on our superconducting microstrip line by considering, in part, the substrate as an arbitrary lossy layer of thickness h, permeability $\mu_0\mu_r(1-j\tan\delta_M)$ and permittivity $\varepsilon_0\varepsilon_r(1-j\tan\delta)$, and, in the other part, the one-layered strip and ground plane (metallization) of thicknesses t_k, permeabilities $\mu_0\mu_{rc_k}(1-j\tan\delta_{MC_k})$ and permittivities $(\varepsilon_0-j\frac{\sigma_k}{\omega})$ with $k = 0$ and 1 respectively. Hence, the expressions of the permittivity and permeability functions in space domain become:

$$\begin{aligned}\varepsilon(x,y,z) = {} & \varepsilon_0-\sum_{k=0}^{1}j\frac{\sigma_k}{\omega}\cdot P_{W/2}[y\cdot\delta(k)]\cdot P_{t_k/2}\left[z-\frac{t_k}{2}+(1-\delta(k))\cdot(t_k+h)\right] \\ & +\varepsilon_0\varepsilon_r\left(1-\frac{1}{\varepsilon_r}-j\tan\delta\right)P_{h/2}\left[z+\frac{h}{2}\right]\end{aligned} \tag{8}$$

$$\begin{aligned}\mu(x,y,z) = {} & \mu_0+\sum_{k=0}^{1}\mu_0\mu_{rc_k}\left(1-\frac{1}{\mu_{rc_k}}-j\tan\delta_{MC_k}\right)\cdot P_{W/2}[y\cdot\delta(k)] \\ & \cdot P_{t_k/2}\left[z-\frac{t_k}{2}+(1-\delta(k))\cdot(t_k+h)\right] \\ & +\mu_0\mu_r\left(1-\frac{1}{\mu_r}-j\tan\delta_M\right)\cdot P_{h/2}\left[z+\frac{h}{2}\right]\end{aligned} \tag{9}$$

The superconducting strip and ground plane are characterized by their thicknesses and complex conductivity using the two-fluid conductivity model [3,13]:

$$\sigma(T) = \sigma_n\left(\frac{T}{T_c}\right)^a - j\frac{1}{\omega\mu_0\lambda_L^2} \tag{10}$$

where σ_n is the normal state conductivity at the closest value of temperature greater than the critical temperature T_c, λ_L is the penetration depth of the magnetic field in the superconductor called London length expressed as follows:

$$\lambda_L(T) = \frac{\lambda_0}{\sqrt{1-(\frac{T}{T_c})^b}} \tag{11}$$

with λ_0 is the penetration depth at $T = 0$ K.

There are two models which give the values of a and b in the expressions (10) and (11). The first one is London model, based on the two-fluid model, valid for the low critical temperature superconducting.

Considering the thermodynamic constraints, this model requires the following value of a and b:

$$a = b = 4 \tag{12}$$