Physica C 471 (2011) 188–192

Contents lists available at ScienceDirect

Physica C

journal homepage: www.elsevier.com/locate/physc

Interband superconductivity in spin-polarized subbands

N. Kristoffel [a,*], K. Veende [b]

[a] Institute of Physics, University of Tartu, 142 Riia Street, 51014 Tartu, Estonia
[b] Institute of Physics, University of Tartu, 4 Tähe Street, 51010 Tartu, Estonia

ARTICLE INFO

Article history:
Received 25 August 2010
Received in revised form 22 October 2010
Accepted 23 December 2010
Available online 29 December 2010

Keywords:
Interband pairing
Spin polarization

ABSTRACT

Singlet superconductivity in spin-polarized subbands, induced by interband pairing interaction, is investigated. A Suhl-type mean-field Hamiltonian is used. The necessary attractive coupling can induce pairing correlations also for the vanishing subbands mismatch shift. The behaviour of superconducting characteristics over the system parameters space is calculated and illustrated. Varying of the pairing strength displays also global phases with inclined metastable superconducting or normal states. First order phase transitions can be expected.

1. Introduction

Superconductivity and ferromagnetism have been regarded as competing orderings during a considerable time. Singlet Cooper pairs are expected to be destroyed by the opposite action of the magnetic field on spin partners. The superconductivity can persist in the fields which do not exceed the upper critical field of the material. Generally the negative conclusion about the coexistence of the (singlet) superconductivity and ferromagnetism has been drawn basing on highly simplified models and restricted material science knowledge.

The whole problem of superconductivity vs. ferromagnetism has afresh appeared with the discovery of novel superconductors of complex structure [1]. As particular examples heavy fermion compounds [2], borocarbides [3] and iron arsenides [4] can be mentioned. Often magnetic and superconducting electrons belong to different subsystems of the material. It has been shown that an inhomogenous triplet superconducting order can appear in the field of magnetic impurities [5,6]. The triplet pairing is expected to be more stable against the magnetic field action and has been widely investigated in various connections [7–10]. The antiferromagnetic ordering borders the superconductivity or even is coexisting with the superconductivity in a number of cases. Complex systems which show both the superconducting and the magnetic ordering possess correspondingly abundant electronic spectra and one enters naturally [7] the area of the multiband superconductivity [11,12].

In the presence of a ferromagnetic field the Fermi surfaces for opposite spin directions do not coincide. There have been numerous approaches to investigate the singlet superconductivity under such circumstances [13–18]. The stability of the found phases with pairing remains elusive in a number of cases. In [17,18] the singlet pairing of two electrons mediated by localized (impurity) spins has been investigated. In this case an attractive interaction on the whole Fermi sea can presumably induce the s-wave superconductivity.

In the present communication, by using a model related to [17], the behaviour of singlet superconducting characteristics with varying pairing conditions is investigated and illustrated. Two spin-polarized subbands of a common origin, shifted on the energy scale by a magnetic action (impurities, other structural elements of the sample, etc.), are proposed to be coupled by an interband pairing interaction. In the presence of the appropriate attractive coupling leading to the formation of pairs of interband constitution the singlet superconductivity (under obvious conditions) becomes possible. This conclusion holds also for the absence of the spin polarization and the background of the Suhl-type interaction constant becomes important. On the route of the system from the normal to the superconducting state with improved pairing conditions the inclined metastable orderings of the opposite nature become possible with expected first order phase transitions.

2. The model Hamiltonian

The Hamiltonian of a two-band superconductor with the interband pairing channel H_i can be written in the form

$$H = H_0 + H_i, \tag{1}$$

* Corresponding author.
E-mail address: kolja@fi.tartu.ee (N. Kristoffel).

0921-4534/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.physc.2010.12.014

where

$$H_0 = \sum_{\vec{k},s} \epsilon_{1\vec{k}} a^+_{1\vec{k}s} a_{1\vec{k}s} + \sum_{\vec{k},s} \epsilon_{2\vec{k}} a^+_{2\vec{k}s} a_{2\vec{k}s}, \quad (2)$$

$$H_i = \sum_{\vec{k},\vec{\xi}} W(\vec{k},\vec{\xi}) \left[a^+_{1\vec{k}\uparrow} a^+_{2-\vec{k}\downarrow} + a^+_{2\vec{k}\uparrow} a^+_{1-\vec{k}\downarrow}\right] \left[a_{1-\vec{\xi}\downarrow} a_{2\vec{\xi}\uparrow} + a_{2-\vec{\xi}\downarrow} a_{1\vec{\xi}\uparrow}\right]. \quad (3)$$

The electron band energies $\epsilon_{1,2}$ in (2) are counted from the chemical potential (μ). Usual designations are used. The interband pairing interaction is characterized by the constant W and it creates singlet pairs formed from the particles of different subbands. Further, a system with spin-polarized subbands with shifted band energies will be considered. Consequently, we attribute the electronic a-operators to the spin-up subband and the b-operators to the spin-down subband. The interaction operator H_i obeys the Suhl [17] form

$$H_i = \sum_{\vec{k},\vec{\xi}} W(\vec{k},\vec{\xi}) a^+_{\vec{k}} b^+_{-\vec{k}} b_{-\vec{\xi}} a_{\vec{\xi}}. \quad (4)$$

In the mean-field approximation one introduces the superconducting gap parameters

$$\Delta_1(\vec{\xi}) = \sum_{\vec{k}} W(\vec{k},\vec{\xi}) \langle a^+_{\vec{k}} b^+_{-\vec{k}} \rangle,$$

$$\Delta_2(\vec{\xi}) = \sum_{\vec{k}} W(\vec{k},\vec{\xi}) \langle b_{-\vec{k}} a_{\vec{k}} \rangle, \quad (5)$$

and (4) is simplified to

$$H_i = \sum_{\vec{k}} \left[\Delta_1(\vec{k}) b_{-\vec{k}} a_{\vec{k}} + \Delta_2(\vec{k}) a^+_{\vec{k}} b^+_{-\vec{k}}\right]. \quad (6)$$

In what follows we describe the spin-polarization in the simplest way as the rigid shift of the band energies $\vec{\xi}_{\vec{k}\uparrow} = \epsilon_a(\vec{k}) + \mu$ and $\vec{\xi}_{\vec{k}\downarrow} = \epsilon_b(\vec{k}) + \mu$ by d in favour of the spin-up (a) band

$$\vec{\xi}_{\vec{k}\uparrow} = d + \vec{\xi}_{\vec{k}\downarrow}. \quad (7)$$

For simplicity we will use a momentum-independent pairing interaction constant W. The sign and correspondingly the physical origin of this interaction will be of significance. In comparable approaches [13–18] one does not specify the nature of W (4-fermion attraction; unspecified electron–electron attraction; pair coupling strength, etc.). In the case of charge-driven interaction a Coulomb-type integral corresponds to the operator form (4). In multiband approaches with the pairs formed from the same band particles the analogon of W is of an exchange (pair-transfer) type, and its sign is free for the creation of pairing [19].

3. The Green's functions and the necessary averages

The problem with the interaction (5) is exactly solvable – the chains of the equations for the two-time Green's functions [20] are automatically broken. The Fourier-transforms of the functions defined on the operators shown in the brackets read

$$G(a_{\vec{k}}; a^+_{\vec{k}}) = \frac{1}{2\pi}(E + \epsilon_{b\vec{k}}) A^{-1}_{\vec{k}}(E),$$

$$G(b^+_{-\vec{k}}; a^+_{\vec{k}}) = \frac{1}{2\pi} \Delta_1 A^{-1}_{\vec{k}}(E), \quad (8)$$

$$G(a^+_{\vec{k}}; a_{\vec{k}}) = \frac{1}{2\pi}(E - \epsilon_{b\vec{k}}) B^{-1}_{\vec{k}}(E),$$

$$G(b_{-\vec{k}}; a_{\vec{k}}) = -\frac{1}{2\pi} \Delta_2 B^{-1}_{\vec{k}}(E), \quad (9)$$

$$G(b^+_{\vec{k}}; b_{\vec{k}}) = \frac{1}{2\pi}(E - \epsilon_{a\vec{k}}) A^{-1}_{\vec{k}}(E),$$

$$G(a_{-\vec{k}}; b_{\vec{k}}) = \frac{1}{2\pi} \Delta_2 A^{-1}_{\vec{k}}(E), \quad (10)$$

$$G(b_{\vec{k}}; b^+_{\vec{k}}) = \frac{1}{2\pi}(E + \epsilon_{a\vec{k}}) B^{-1}_{\vec{k}}(E),$$

$$G(a^+_{-\vec{k}}; b^+_{\vec{k}}) = -\frac{1}{2\pi} \Delta_1 B^{-1}_{\vec{k}}(E). \quad (11)$$

For the resonance denominators in the Green's functions the abbreviations

$$A(E) = (E - E_A(+))(E - E_A(-)),$$
$$B(E) = (E - E_B(+))(E - E_B(-)), \quad (12)$$

are used where the quasiparticle excitation energies equal

$$E_{A\vec{k}}(\pm) = \frac{1}{2}(d \pm D_{\vec{k}}),$$
$$E_{B\vec{k}}(\mp) = -\frac{1}{2}(d \mp D_{\vec{k}}). \quad (13)$$

Here

$$D_{\vec{k}} = \left[(\epsilon_{a\vec{k}} + \epsilon_{b\vec{k}})^2 + 4\Delta_1\Delta_2\right]^{1/2}. \quad (14)$$

The necessary statistical averages follow from the calculated Green's functions by the analytic continuation [20] as

$$\langle a_{\vec{k}} a^+_{\vec{k}} \rangle = D^{-1}[(E_A(+) + \epsilon_b)F(-E_A(+)) - (E_A(-) + \epsilon_b)F(-E_A(-))],$$
$$\langle a^+_{\vec{k}} a_k \rangle = D^{-1}[(E_A(+) + \epsilon_b)F(E_A(+)) - (E_A(-) + \epsilon_b)F(E_A(-))], \quad (15)$$
$$\langle b_k b^+_k \rangle = D^{-1}[(E_A(+) - \epsilon_a)F(E_A(+)) - (E_A(-) + \epsilon_a)F(E_A(-))],$$
$$\langle b^+_k b_k \rangle = D^{-1}[(E_A(+) - \epsilon_a)F(-E_A(+)) - (E_A(-) - \epsilon_a)F(-E_A(-))]. \quad (16)$$

The anomalous averages characterizing the presence of superconducting correlations are of the form

$$\langle a^+_k b^+_{-k} \rangle = \Delta_1 D^{-1}[F(E_A(+)) - F(E_A(-))],$$
$$\langle b^+_{-k} a^+_{\vec{k}} \rangle = \Delta_1 D^{-1}[F(-E_A(+)) - F(-E_A(-))], \quad (17)$$
$$\langle a_k b^+_{-k} \rangle = \Delta_2 D^{-1}[F(-E_A(+)) - F(-E_A(-))],$$
$$\langle b^+_{-k} a_k \rangle = \Delta_2 D^{-1}[F(E_A(+)) - F(E_A(-))]. \quad (18)$$

The Fermi distribution function is designated by ($\Theta = k_bT$)

$$F(x) = [e^{\frac{x}{\Theta}} + 1]^{-1}. \quad (19)$$

4. Superconducting characteristics

According to (5), (17) and (18) the system determining the superconducting gap parameters is of the form

$$\Delta_{1,2} = \Delta_{1,2} W \sum_{\vec{k}} D^{-1}_{\vec{k}} [F(E_{A\vec{k}}(+)) - F(E_{A\vec{k}}(-))]. \quad (20)$$

The solutions $\Delta_{1,2} = 0$ of (20) correspond to the normal phase. The remaining equation ($\Delta_{1,2} \neq 0$).

$$1 = W \sum_{\vec{k}} D^{-1}_{\vec{k}} [F(E_{A\vec{k}}(+)) - F(E_{A\vec{k}}(-))] \quad (21)$$

allows one to find only the composite quantity $\delta^2 = 4\Delta_1\Delta_2$, i.e. the Δ^2, as expected from (5). In general this equation determines the nonequilibrium gap parameter as a measure of pairing correlations strength between the particles on the two Fermi surfaces. By substituting $\Delta = 0$ into (21) one reaches to the equation for the superconducting transition temperature (T_c).

Because $E_A(+) > 0$ and $E_A(-) < 0$ one can see that Eq. (21) can be fulfilled only in the case of a negative $W < 0$, i.e. an attractive pairing coupling between the spin subbands. The renormalized band populations sum up to ($\epsilon_{\vec{k}} = \epsilon_{a\vec{k}} + \epsilon_{b\vec{k}}$)

$$\langle a^+_{\vec{k}} a_{\vec{k}} \rangle + \langle b^+_{\vec{k}} b_{\vec{k}} \rangle = 1 - \frac{\epsilon_{\vec{k}}}{2D_{\vec{k}}} \left[\tanh\frac{D_{\vec{k}} + d}{4\Theta} + \tanh\frac{D_{\vec{k}} - d}{4\Theta}\right]. \quad (22)$$

This expression is transformed into the sum $F(\epsilon_a/\Theta) + F(\epsilon_b/\Theta)$ for the normal phase.

We are investigating the superconducting properties of the present model by using constant ($2D$) densities of states $\rho = \rho_{a,b}$