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# The model of Josephson junction arrays embedded in resonant cavities and the phase-locking properties

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ABSTRACT

We propose a new model of a Josephson junction array embedded in a resonant cavity. The model considers the excitation of the resonance mode by the array and the influence of the resonance mode on the array. The phase-locking properties of the junction array are investigated in the frame of the model. Resonant steps in the current–voltage characteristics due to the interaction between the array and the resonant cavity and many other features of the phase-locking behaviors have been produced. The results include the influence of the quality factor, the strength of the coupling, as well as the junction number of the array on the phase-locking properties. The model can successfully explain many features of the real situation and provide useful new predictions.

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## 1. Introduction

The use of Josephson junction arrays as millimeter and sub-millimeter wave sources is one of the most promising applications in the superconducting electronics [1–5]. The challenge of such applications is that the high-power coherent radiation is only generated when the junctions in the array is phase-locked. The key feature of the phase-locking dynamics is the competition between the disorder (i.e., variations in the junction resistances and critical currents) and the coupling between the junctions. When there is coupling between the junctions, the nonlinear interactions cause the elements to oscillate at shifted frequencies. So, populations of coupled Josephson junctions can spontaneously synchronize to a common frequency, despite differences in their parameters. To realize the phase-locking state, many methods/models have been developed, among of which is imbedding series Josephson junction array in a resonant cavity [6]. In order to understand experimental results and provide useful predictions, simulations are necessary. In the Refs. [4,7–10], studies are based on the Josephson junction array shunted by an *LCR* series resonance circuit directly. However, this model can not reflect the practical situations. For example, the anti-phase locking state of the junctions is obtained in the Ref. [8], but this is not correct and there is no experimental result to support it. The AC voltage generated by the array will be bypassed when its frequency is equal to the resonance frequency of the *LCR* circuit. It will lead to lower AC voltage so that the anti-phase locking state is apt to appear. Ref. [10] adopts this model and points out, in this model, there is no threshold of the junction number for phase-locking state; the experimental below-threshold region behavior could not be reproduced. This suggests that there is an intrinsic difference between the experiment of an array imbedded in a resonant cavity and the directly shunted *LCR* model.

On the basis of the directly *LCR* shunted junction array model, we put forward a new model, which takes the interaction between the array and the cavity modes into account. It can also reflect the quality factor of the cavity and strength of the coupling between the junction and the cavity. The model is rather simple, but it is very useful. It can successfully explain many features the directly *LCR* shunted junction array model can not. The paper will report our method and simulation results.

## 2. Formulation

Shown in Fig. 1 is the schematic circuit of a Josephson junction array shunted by an *LCR* resonance circuit adopted in the Refs. [4,7–10].

*L*, *C* and *R* are the lumped elements forming the resonator. For an array consisting of *n* resistively and capacitively shunted junctions, the governing circuit equations are:

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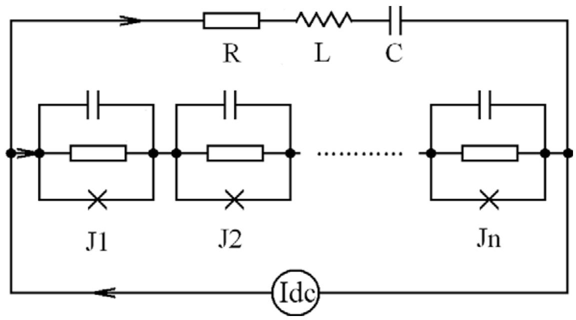


**Fig. 1.** Schematic of a Josephson junction series array shunted by a lumped resonant circuit.

$$V_i = \frac{\hbar}{2e}\frac{d\varphi_i}{dt} \quad (i = 1, 2, \cdots, n) \tag{1.1}$$

$$\frac{V_i}{R_i} + C_i\frac{dV_i}{dt} + I_{ci}\sin\varphi_i + \frac{dq}{dt} = I_{dc} \quad (i = 1, 2, \ldots, n) \tag{1.2}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = \sum_{i=1}^{n} V_i \tag{1.3}$$

where $\varphi_i$ is the wave-function phase difference across the $i$th junction, $R_i$ is the junction resistance, $I_{ci}$ is the junction critical current. $q$ is the charge on the capacitor $C$, $\hbar$ is the Planck's constant divided by $2\pi$, and $e$ is the elementary charge. The voltage drop across the $i$th junction is $V_i$. Eqs. (1.1) and (1.2) are simply the equation of motion for a Josephson junction biased with a dc current $I_{dc}$ and the current flowing through the *LCR* circuit. Eq. (1.3) is the equation of motion for the series *LCR* circuit subject to a voltage that is the sum of the voltages of the Josephson elements. As mentioned above, this model can not exactly reflect the situations of a Josephson junction array in a resonant cavity. In order to simulate the array in resonant cavity, we propose the new model:

$$V_i = \frac{\hbar}{2e}\frac{d\varphi_i}{dt} \quad (i = 1, 2, \ldots, n) \tag{2.1}$$

$$\frac{V_i}{R_i} + C_i\frac{dV_i}{dt} + I_{ci}\sin\varphi_i = I_{dc} + k_1\frac{dq}{dt} \quad (i = 1, 2, \ldots, n) \tag{2.2}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = k_2\sum_{i=1}^{n} V_i \tag{2.3}$$

Compared with the equations of (1.1), (1.2), and (1.3), there are two differences in the equations of (2.1), (2.2), and (2.3). First, the current flowing through the *LCR* circuit, $dq/dt$, is multiplied by a coefficient $k_1$ and shifted to the right hand side of Eq. (2.2). Second, the total voltage across the array $\sum_{i=1}^{n} V_i$ in Eq. (2.3) is multiplied by another coefficient $k_2$. The reasons are as follows. In the real situation of an array in a cavity, the voltage of the array will excite the cavity modes. The higher the total AC voltage of the array and the stronger the coupling between the array and the cavity, the stronger the oscillation modes excited. The coupling between them is implied in $k_2$ of Eq. (2.3), and the quality factor ($Q$) of the cavity is tuned by the $R$ of the *LCR* circuit. When there is oscillating mode in the cavity, there is inductive current on the array, which impacts the behavior of the array in return. This process is characterized by Eq. (2.2), where the coupling is implied in $k_1$. The stronger the coupling, the more significant the impact from the cavity oscillating mode on the array. The product of the coefficients $k_1$ and $k_2$ reflects the strength of the coupling, so we defined $k = k_1k_2$ as the coupling coefficient. The term representing the microwave current is on the right hand side instead of left, because it is required that the field and current should parallel to each other in order to avoid anti-phase locking.

An array of 1000 junctions is used for the calculation and divided into five groups. The junction parameters of the same group are identical for simplicity. The 1st group consists of 500 junctions and the critical current $I_{c1}$ is 0.3 mA, junction resistance $R_1$ is 3 Ω and junction capacitance $C_1$ is 0.05 pF. The McCumber parameter is about 0.41 (generally speaking, for HTS bi-crystal junctions, the McCumber parameter is less than one). The 2nd group and the 3rd group both consist of 150 junctions. To bring in the parameter spreads, the parameters of the 2nd group are set as: $I_{c2} = (1 + err1)\ I_{c1}$, $R_2 = R_1/(1 + err1)$, and $C_2 = (1 + err1)\ C_1$; the parameters of the 3rd group are set as $I_{c3} = I_{c1}/(1 + err1)$, $R_3 = (1 + err1)\ R_1$, and $C_3 = C_1/(1 + err1)$. $err1$ ($err1 = 0.01$) is a small positive number to donate the parameter spreads between the three groups. The 4th group and the 5th group both consist of 100 junctions. $I_{c4} = (1 + err2)\ I_{c1}$, $R_4 = R_1/(1 + err2)$, $C_4 = (1 + err2)\ C_1$, $I_{c5} = I_{c1}/(1 + err2)$, $R_5 = (1 + err2)\ R_1$, and $C_5 = C_1/(1 + err2)$. $err2$ ($err2 = 0.02$) is larger than $err1$ and is used to donate the errors between the groups of 1st, 4th and 5th. The inductance in Eq. (2.3) $L = 1$ pH, the capacitance $C = 0.5$ pF. The resonance frequency is 225 GHz. The resistance $R = 0.01$ Ω, which is a sign of the loss. The larger the resistance, the smaller the quality factor. When the resistance $R$ is equal to 0.01 Ω, the corresponding quality factor $Q$ is 141.

Eqs. (2.1)–(2.3) are solved using a 4th order Runge-Kutta integration algorithm and the sum of the junction's voltage is recorded and analyzed.

## 3. Results and discussion

Fig. 2 shows the simulated *I–V* characteristic curve. It is clear that there is a resonant step at the voltage of 0.47 V. If the voltage across 1000 junctions is fixed at 0.47 V, the corresponding Josephson frequency is 227.3 GHz which is in accordance with the resonance frequency of the *LCR* circuit. It is clear that the step is distorted, but the distortion is not caused by the calculation accuracy. Calculations with higher accuracy show the same phenomenon. It may be caused by complex dynamics and will be researched in further work.

To investigate the different dynamics of the Josephson junction array when it is or not biased on the step, we set $I_{dc}$ at 0.33 mA and 0.34 mA, respectively, and record the time domain voltage waveform. Fig. 3 shows the voltage waveform of the array when $I_{dc}$ is 0.33 mA. The coherent wave could be seen, which indicates the phase-locking state of the array. Fig. 4 shows the voltage waveform of the array when $I_{dc}$ is 0.34 mA, where the coherent wave is not obtained, because the junctions fail to phase-lock and the phase relation is random.

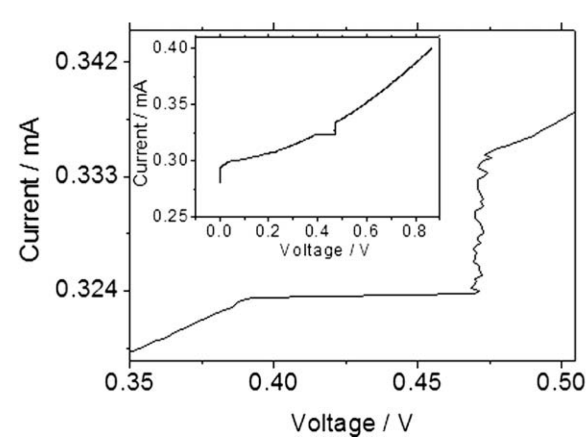


**Fig. 2.** *I–V* characteristic curve of the array coupled with a resonator.

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