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Josephson current in a graphene SG/ferromagnetic barrier/SG junction

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ABSTRACT

The Josephson current passing through a $SG_1/F_B/SG_2$ graphene junction, where SG and F_B are those parts of a graphene layer which are induced into the superconducting state and into the ferromagnetic state, respectively, and where the small thickness of the F_B layer L is studied. The ferromagnetic barrier strength is taken to be given by $\chi_H \sim HL/hv_F$, where H is the strength of the exchange energy and $v_F \sim 10^6$ m/s is the Fermi velocity of quasiparticles. The eigenstates of the relativistic quasiparticles in the graphene are taken to be the solutions of the Dirac Bogoliubov-de Gennes equations. It is found that the energy levels of the Andreev bound states for the Weyl–Dirac particles in the $SG_1/F_B/SG_2$ junction are independent of the direction of the spins and that they depend on the strength of ferromagnetic barrier potential. The critical supercurrent is seen to vary in an oscillatory (periodic) manner as χ_H is varied. The oscillatory behavior of the critical supercurrent carried by the Cooper pairs formed by massless the Weyl–Dirac particles is different from the behavior of the supercurrent carried by the Cooper pairs formed by non-relativistic particles in a conventional SC/FI/SC (FI being a ferromagnetic insulator) junction. In those types of junctions, the supercurrent does not exhibit a similar oscillatory dependence.

1. Introduction

The recent fabrication of "graphene", a monolayer of graphite [1], has attracted much attention and has lead to many studies of its electronic properties. Of special interest is the fact that the energy spectrum of the quasiparticles moving in the stable two-dimensional (2D) layer of graphite (graphene) is not given by the Newtonian relationship, $E = p^2/2m$, but is instead given by $E = pc_{\text{eff}}$, where c_{eff} is the effective speed of light and is $\sim 10^6$ m/s, the Fermi velocity (v_F) in graphene [2,3]. This energy spectrum is that of the relativistic particles of zero mass, the Weyl–Dirac particles. These particles are described by the 2D Dirac equation for massless particles. The behavior of the quasiparticles in graphene should mimic the behavior of the relativistic massless electrons. These types of quasiparticles when injected into an insulating layer at normal incidence to the interface will have transmission probability amplitudes similar to those seen when quasiparticles tunnel through a junction which has no insulator located at the interface [4]. This is called "Klein paradox" [5].

The relativistic nature of the quasiparticles in the 2D graphene layer leads to the scattering of these particles at a normal graphene (NG)/superconducting graphene (SG) interface to be different at that from a normal metal (NM)/conventional superconductor (SC) interface. The tunneling of non-relativistic electrons (holes) into the SC region on the RHS of a NM/SC interface leads to a retro reflection of the Andreev reflected holes (electrons) while the tunneling of 2D relativistic electrons (holes) of energies above the Fermi energy into the RHS of a NG/SG interface gives rise to specular Andreev reflections of the relativistic particles [6].

The differences in the Andreev reflections at the interfaces in conventional superconductor nano sandwiches and graphene based nano sandwiches will lead to differences in the behaviors of the two types of sandwiches. The supercurrents in graphene based superconducting devices, such as SG/NG/SG junctions, have been studied both theoretically [7–9] and experimentally [10]. Most of the studies of the transport properties of conventional superconductor sandwiches have been based on the Blonder, Tinkham and Klapwijk (BTK) approach [11]. In this approach, the probability amplitudes for the transmission of the different particles on one side of the junctions to the other side are determined by applying the appropriate boundary conditions at the interfaces in the sandwiches. For the cases involving graphene based sandwiches, the BTK approach is still used. However, the boundary conditions applied to the relativistic particles are different from those applied to the non-relativistic particles.

In this paper, we study the supercurrent flowing through a graphene based SG/FG/SG junction in which the graphene is

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induced in the superconducting state and ferromagnetic state by the proximity effect tunneling of the superconducting order parameter or the magnetic order parameter of a superconductor or of a ferromagnet placed above the graphene, respectively [12,13]. The junction we consider in this study consists of two SG layers separated by a single FG layer of thickness L (see Fig. 1). Ferromagnetic graphene can also be achieved by doping the graphene. We have applied the boundary conditions appropriate for this junction to the solutions of the Dirac Bogoliubov-de Gennes equations to obtain the energy levels of the Andreev bound states. An expression for the supercurrent I_C is obtained through a differentiation of the expression for Andreev bound state energy.

The expression for I_C is then numerically evaluated for incremental changes of the values of several parameters characterizing the junction. The critical current is the maximum value of the phase dependent supercurrent as the phase difference between the two SG layers is varied from 0 to $\pi/2$ [14,15]. The supercurrent due to the Klein effect is investigated by changing the angle of incidence of the quasiparticles in the SG layer. We find that the Andreev energy levels for the relativistic particles tunneling through the barrier are independent of the spin direction. Our numerical evaluations of the supercurrent do not reveal a 0–π phase state transition. They do, however, reveal an oscillatory behavior of the supercurrent as the ferromagnetic barrier strength χ_H is varied unlike that seen in conventional SC/ferromagnetic barrier/SC junctions. The oscillatory behaviors seen could either be due to changes in the value of the ferromagnetic exchange field or to changes in the thickness of the FG layer.

In our derivations, we have assumed that the Fermi energy of ferromagnetic graphene, E_{FF}, is zero which would be possible if the FG was in an under doped state and that the Fermi energy of SG, E_{FS}, is much larger than the superconducting gap. More importantly, we assume that a strong ferromagnetic exchange field H exists within the thin FG layer. This field leads to a ferromagnetic barrier, whose strength is given by $\chi_H \sim HL/\hbar v_F$, where H is the strength of the exchange energy, which is taken to be large and where $v_F \sim 10^6$ m/s is the Fermi velocity of the quasiparticles in graphene, to form at the interface. We then simulate the effects of varying the value of the exchange field within the FG layer by carrying out numerical calculations.

The motivation for studying the effects of varying the values of different parameters on the behaviors of different graphene based junctions by using numerical simulations are that insights into possible new applications of graphene based electronics might be gained. Yokoyama [16] showed that the spin currents could be controlled by varying the gate voltage and magnetic field in the FG layer. Cresti [17] has suggested the possible fabrication of a graphene-based current nanoswitch. A graphene based field effect transistor whose observed behaviors were in quantitative agreement with a drift-diffusion model of spin transport has been fabricated by Józsa et al. [18]. Gang Li et al. [19] have fabricated a graphene-based p–n–p junction with contactless top gate.

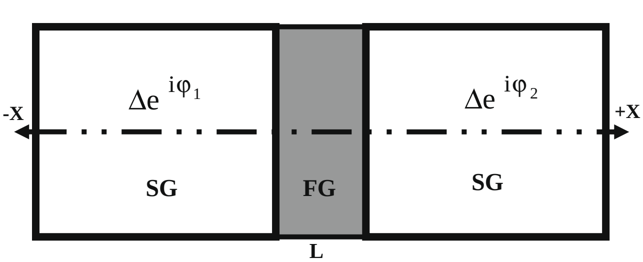

Fig. 1. The schematic illustration of a SG/FG/SG junction. The thickness of FG is L and the superconducting phases for the left SG and the right SG are φ_1 and φ_2, respectively.

2. Andreev energy levels

The particles propagating in the 2D honeycomb array of carbon ions (graphene) mimic the behavior of relativistic particles of zero mass. These particles would then be the solutions of the Dirac Bogoliubov-de Gennes equations

$$\begin{bmatrix} v_F\hbar\vec{\sigma}\cdot\hat{\vec{k}} + U(x,y) - H_\sigma(x,y) & \Delta(x,y) \\ \Delta^*(x,y) & -v_F\hbar\vec{\sigma}\cdot\hat{\vec{k}} - U(x,y) - H_\sigma(x,y) \end{bmatrix} \psi_\sigma(x,y) = E,\psi_\sigma(x,y), \tag{1}$$

where $U(x,y) = E_{FF}\Theta(-x)\Theta(x+d) - E_{FS}\Theta(-x-d) + E_{FS}\Theta(x)$, with E, E_{FF}, E_{FS} and $\Theta(x)$ being the excitation energy, the Fermi energy of the FG, the Fermi energy of the SG and the Heaviside step function, respectively. The superconducting order parameter is defined as $\Delta(x,y) = \Delta e^{i\varphi_1}\Theta(-x-d) + \Delta e^{i\varphi_2}\Theta(x)$, with $\varphi_{1,2}$ being the superconducting phases in the left SG and right SG, respectively. For the spin dependent potential of system, $H_\sigma(x,y) = \eta_\sigma H\Theta(-x)\Theta(x+d)$, where H is the exchange energy in the FG barrier and $\eta_\sigma = \eta_{\uparrow(\downarrow)}$ is $1(-1)$. The Pauli spin matrix is $\vec{\sigma} = \langle\sigma_x, \sigma_y\rangle$ and the wave vector operator of the quasiparticle is $\hat{\vec{k}} = \langle -i\partial/\partial x, -i\partial/\partial y\rangle$. The parallel component k_y is equal to $2n\pi/d$ where d is the width of Josephson's junction and n is the quantum number of the bounded vibrating state representation of the eigenstates of a Weyl–Dirac particle.

The quasiparticle wave functions, in which $k_{FS}\xi_s \gg 1$ or $E_{FS} \gg \Delta$ with superconducting coherent length being defined as $\xi_s = \hbar v_F/\Delta$ can be thought of as a superposition of the spin dependent four component spinor state solutions of Eq. (1), i.e., as [8]

$$\psi_\sigma(x \leqslant -L, y) = e^{ik_y y}\left(a_\sigma \begin{bmatrix} 1 \\ e^{i\theta} \\ e^{-i(\varphi_1-\beta)} \\ e^{i(-\varphi_1+\beta+\theta)} \end{bmatrix} e^{ik_s x+\kappa x} + b_\sigma \begin{bmatrix} 1 \\ -e^{-i\theta} \\ e^{-i(\varphi_1+\beta)} \\ -e^{-i(\varphi_1+\beta+\theta)} \end{bmatrix} e^{-ik_s x+\kappa x} \right), \tag{2}$$

$$\psi_\sigma(-L \leqslant x \leqslant 0, y) = e^{ik_y y}\left(g_{1\sigma} \begin{bmatrix} 1 \\ e^{i\alpha_e} \\ 0 \\ 0 \end{bmatrix} e^{ik_{F+}x} + g_{2\sigma} \begin{bmatrix} 1 \\ -e^{-i\alpha_e} \\ 0 \\ 0 \end{bmatrix} e^{-ik_{F+}x} + g_{3\sigma} \begin{bmatrix} 0 \\ 0 \\ 1 \\ e^{-i\alpha_h} \end{bmatrix} e^{ik_{F-}x} + g_{4\sigma} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -e^{i\alpha_h} \end{bmatrix} e^{-ik_{F-}x} \right) \tag{3}$$

and

$$\psi_\sigma(0 \leqslant x, y) = e^{ik_y y}\left(c_\sigma \begin{bmatrix} 1 \\ e^{i\theta} \\ e^{-i(\varphi_1+\beta)} \\ e^{i(-\varphi_1-\beta+\theta)} \end{bmatrix} e^{ik_s x-\kappa x} + d_\sigma \begin{bmatrix} 1 \\ -e^{-i\theta} \\ e^{-i(\varphi_1-\beta)} \\ -e^{-i(\varphi_1-\beta+\theta)} \end{bmatrix} e^{-ik_s x-\kappa x} \right). \tag{4}$$

Here,

$$\sin[\alpha_{e(h)}] = \hbar v_F k_y/[E + \eta_\sigma H + (-)E_{FF}], \quad \sin[\theta] = \hbar v_F k_y/E_{FS}$$

$$k_{F\pm} = \sqrt{\left(\frac{E+\eta_\sigma H \pm E_{FF}}{\hbar v_F}\right)^2 - k_y^2}, \quad k_s = \sqrt{\left(\frac{E_{FS}}{\hbar v_F}\right)^2 - k_y^2}$$

$$\kappa^{-1} = (\hbar v_F)^2 k_s/[E_{FS}\Delta\sin(\beta)], \quad \beta(|E| < \Delta) = \mathrm{Arccos}(E/\Delta),$$