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Andreev multiple reflection and current in a junction with a quantum dot embedded in a superconducting ring

Ben-Ling Gao [a,c,*], Qing-Qiang Xu [b,c], Shi-Jie Xiong [c]

[a] Department of Physics, Huaiyin Institute of Technology, Huaian 223003, People's Republic of China
[b] Department of Physics, Xuzhou Normal University, Xuzhou 221009, People's Republic of China
[c] National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing 210093, People's Republic of China

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Abstract

We investigate the properties of Andreev multiple reflection and supercurrent in a normal-metal junction with a quantum dot embedded in a mesoscopic superconducting ring. The quantum dot is in the Coulomb blockade regime and contains several energy levels. The supercurrent through the junction is created by an external magnetic flux threaded through the ring. The supercurrent is related to the multiple Andreev reflection at interfaces between the superconductor and the normal metal, which is influenced by the Coulomb blockade on the quantum dot. Using a tight-binding model with Bogoliubov-de Gennes pairing potential, the supercurrent versus the external flux is calculated. By switching on the gate voltage applied on the dot, the supercurrent shows a series of peaks and valleys for various values of the flux through the ring. The results suggest the strong effect of the quantum dot on the supercurrent in such a structure, and the possible methods of controlling the supercurrent.

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Keywords: Coulomb blockade; Quantum dots; Andreev reflection

1. Introduction

In recent years, the transport of electrons in mesoscopic structures has drawn considerable attention of theoretical and experimental researchers [1]. Among them the structures based on SQUID and junctions between superconductors (S) and normal metals (N) have been especially focused due to possible applications in devices of quantum computing or quantum information [2–4]. In these systems, the Andreev reflection (AR) at S/N junctions [5] plays an important role. Thus, phenomena involving AR have been extensively studied [6–9]. Particularly, properties of S/N junctions with quantum dots have caused considerable attention [6,10–14]. However, up to the date the effect of Coulomb blockade in quantum dots on the supercurrent in these structures have not been considered in details.

In this paper, we investigate the properties of supercurrent in a normal-metal junction with a quantum dot embedded in a mesoscopic superconducting ring, as illustrated in Fig. 1. The supercurrent through the junction is created by an external magnetic flux threaded through the ring. Using a tight-binding model with Bogoliubov-de Gennes pairing potential, we investigate the combined effects of the Andreev multiple reflection, the magnetic flux, the level structure and the Coulomb blockade of the dot on the supercurrent. The supercurrent versus the external flux is calculated. By switching on the gate voltage on the dot, the supercurrents shows a series of peaks and valleys for given value of flux. The results suggest the strong effect

* Corresponding author. Address: Department of Physics, Huaiyin Institute of Technology, Shanghai road, Huaian 223003, China. Tel.: +86 516 83452879.
E-mail address: xuqing_qiang@sina.com (B.-L. Gao).

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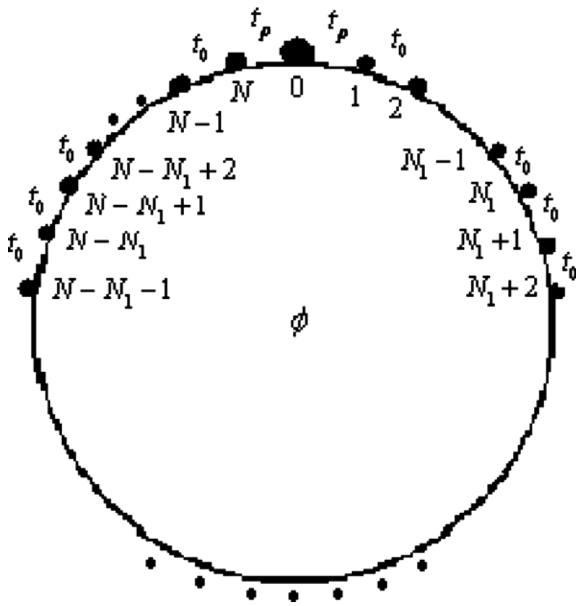

Fig. 1. The schematic figure of the system.

of the quantum dot on the supercurrent, and the possible methods of controlling the supercurrent in such structures.

2. Basic model and formulae

The Hamiltonian of the system can be expressed as

$$H = H_S + H_N + H_D + H_T \quad (1)$$

where H_S, H_N, H_D, and H_T represent the sup-Hamiltonians of the superconductor part, the normal metal, the quantum dot, and the coupling between the dot and the normal metal, respectively. In a tight-binding model they are written as:

$$H_S = \sum_{m=N_1,}^{N-N_1} \sum_{\sigma} t_0 (c_{m,\sigma}^{\dagger} c_{m+1,\sigma} + \text{H.c.}) + \sum_{m=N_1+1}^{N-N_1} \Delta (c_{m,\uparrow}^{\dagger} c_{m,\downarrow}^{\dagger} + \text{H.c.}), \quad (2)$$

$$H_N = \sum_{m=N-N_1+1,}^{N-1} \sum_{\sigma} t_0 (c_{m,\sigma}^{\dagger} c_{m+1,\sigma} e^{i\varphi} + \text{H.c.}) + \sum_{m=1,}^{N_1-1} \sum_{\sigma} t_0 (c_{m,\sigma}^{\dagger} c_{m+1,\sigma} e^{i\varphi} + \text{H.c.}), \quad (3)$$

$$H_D = \sum_{i=1}^{L} \sum_{\sigma} (\xi_i + v_g) c_{0,i,\sigma}^{\dagger} c_{0,i,\sigma} + \frac{e^2}{2C} \left(\sum_{i=1}^{M} \sum_{\sigma} c_{0,i,\sigma}^{\dagger} c_{0,i,\sigma} \right)^2, \quad (4)$$

and

$$H_T = \sum_{i,\sigma} t_p (c_{N,\sigma}^{\dagger} c_{0,i,\sigma} e^{i\varphi} + c_{0,i,\sigma}^{\dagger} c_{1,\sigma} e^{i\varphi} + \text{H.c.}), \quad (5)$$

where the ring consists of $N+1$ sites that are made of one dot (site 0), $2N_1$ sites of normal metal (from site $N-N_1+1$ to site N and from site 1 to site N_1) and $N-2N_1$ sites of superconducting part (from site N_1+1 to site $N-N_1$). Symbols $c_{m,\sigma}^{\dagger}(c_{m,\sigma})$ and $c_{0,i,\sigma}^{\dagger}(c_{0,i,\sigma})$ are creation (annihilation) operators of election with spin σ at site m in the ring (except for the dot) and in level i of the dot, respectively. t_0 is the nearest-neighbor hopping integral in the ring, and t_p is the coupling strength between levels of the dot and the normal metal. Δ is the Bogoliubov-de Gennes pairing potential for the superconducting part. On the dot there are L energy levels ξ_i with $i = 1, 2, \ldots, L$, and they can be shifted by the gate voltage v_g applied on the dot. The charging energy on the dot is characterized by an effective capacitance C. The phase factor φ is related to the magnetic flux ϕ enclosed by the ring with $\varphi = 2\pi\phi/(2N_1\phi_0)$, where $\phi_0 = hc/e$ is the elementary flux quantum. Due to the diamagnetism of the superconductor part, the vector potential induced by the flux only appears in the normal part.

Owing to the Bogoliubov-de Gennes potential, the elections in the ground state are paired to form Cooper pairs in the superconducting part. The superconducting gap is located about the Fermi level. An election or a hole in the normal part with energy within the gap will be Andreev reflected repeatedly at two boundaries between superconducting and normal parts, resulting in Andreev bound states located in the normal part. We suppose that M elections occupy several levels on the dot before an election or a hole tunnels into it originally, which corresponds to state D in the following wave functions. When an election or a hole enters the dot, the number of electrons on the dot becomes $M \pm 1$. The value of M is controlled by the voltage v_g and the interaction of elections in the dot [15]. With the use of the Nambu representation, a quasiparticle state can be expressed as

$$\Phi_1 = \sum_{m=1}^{N} \sum_{\sigma} (a_{m,\sigma} c_{m,\sigma}^{\dagger} + b_{m,-\sigma} c_{m,-\sigma}) |F\rangle, \quad (6)$$

where $|F\rangle$ denotes the Fermi sea. At the same time, the many-body state of the dot can be expressed as:

$$\Phi_2 = \prod_{\{i,\sigma'\}\in D} c_{0,i,\sigma'}^{\dagger} |0\rangle, \quad (7)$$

where $|0\rangle$ denotes the vacuum state. The wave function of the system, including the electrons on the dot and the tunneling electron or hole, can be expressed with products of Φ_1 and Φ_2. As there are other states with the dot out of state Φ_1, we write the general form of the wave function as

$$\Psi = \psi_1 + \psi_2, \quad (8)$$

where ψ_1 represent the component of initial or final state of the tunneling in which the tunneling electron or hole is outside the dot and the electrons on the dot are in their original state, and ψ_2 is the component with the electrons of the dot out of their original state. These two components are described as:

$$\psi_1 = \sum_{m=1}^{N} \sum_{\sigma} (a_{m,\sigma} c_{m,\sigma}^{\dagger} + b_{m,-\sigma} c_{m,-\sigma}) |F\rangle \otimes \prod_{\{i,\sigma'\}\in D} c_{0,i,\sigma'}^{\dagger} |0\rangle, \quad (9)$$

$$\psi_2 = \sum_{i,\sigma} (a'_{i,\sigma} c_{0,i,\sigma}^{\dagger} + b'_{i,-\sigma} c_{0,i,-\sigma}) \prod_{\{i,\sigma'\}\in D} c_{0,i,\sigma'}^{\dagger} |0\rangle. \quad (10)$$