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The interaction of two collinear cracks in a rectangular superconductor slab under an electromagnetic force

Zhi-Wen Gao [a,b,c], You-He Zhou [a,b], Kang Yong Lee [c,*]

[a] Key Laboratory of Mechanics on Western Disaster and Environment, Ministry of Education, PR China
[b] Department of Mechanics and Engineering Science, College of Civil Engineering and Mechanics, Lanzhou University, Lanzhou, Gansu 730000, PR China
[c] School of Mechanical Engineering, Yonsei University, Seoul 120-749, Republic of Korea

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ABSTRACT

The interaction of two collinear cracks is obtained for a type-II superconducting under electromagnetic force. Fracture analysis is performed by means of finite element method and the magnetic behavior of superconductor is described by the critical-state Bean model. The stress intensity factors at the crack tips can be obtained and discussed for decreasing field after zero-field cooling. It is revealed that the stress intensity factor decreases as applied field increases. The crack-tip stress intensity factors decrease when the distance between the two collinear cracks increases and the superconductors with smaller crack has more remarkable shielding effect than those with larger cracks.

1. Introduction

Bulk high-temperature superconductors are developed for applications in power engineering applications such as magnetic bearings, flywheel storage energy systems, and trapped field magnet [1]. Due to brittleness of superconductors' solids, more and more researchers have focused their attention on mechanical analysis of the bulk superconductors. Micro-cracks are always unavoidable, which are associated with the stress induced in the superconducting matrix by the *YBCO* particles, during cool down from the processing temperature [2]. These cracks may affect the mechanical characteristic of superconductor, and limit the use of bulk high-temperature superconductors. Therefore, the fracture analysis of the superconductor is needed for high reliability in accordance with safety evaluations. The flux-pinning-induced magnetostriction model was first described by Ikuta et al. [3]. Johansen analyzed theoretically the electromagnetic force arising from flux pinning on superconductor [4,5]. The crack problems of superconductor have been preliminary studied by some researchers [6–10]. In practical operation, especially for the trapped field magnet, fracture phenomenon is observed in the experiments. Ren et al. [11] present the experiment and theory to study the damage of high trapped field magnet under electromagnetic force. In general, the electromagnetic force is considered as the cause of the fracture in superconductors [11]. To the best of our knowledge, the interaction of collinear cracks for the superconductor crack problem has not been reported in the literatures. The behavior of collinear cracks had been investigated by some researchers in other material [12–16].

The purpose of the present paper is to investigate the interaction of collinear cracks for a long rectangular slab of superconductor by using the finite element method. The magnetic behavior of superconductor is described by the critical-state Bean model [17]. The stress intensity factors at the crack tips can be obtained and discussed for decreasing field after zero-field cooling. The stress intensity factors are chosen as fracture parameters, and the crack tip interaction of the crack position and crack length are revealed by parametric studies on the numerical results of stress intensity factors.

2. Electromagnetic force and numerical analysis model

It is assumed that there are two collinear cracks of length $2a$ along the x-axis in rectangular slab superconductor placed in a magnetic field B_a as shown in Fig. 1. The slab is placed in a parallel magnetic field oriented parallel to the z-axis. The symbol d represents the distance between two collinear cracks. Two collinear cracks are embedded in a cracked superconductor slab with rectangular cross-section (see Fig. 2). The slab is in infinitely long in the z-direction, so that the demagnetization effects can be ignored. The crack is subjected to electromagnetic forces arising from flux pinning, as shown in Fig. 2. Additionally, we seek the solutions of

* Corresponding author. Tel./fax: +82 2 2123 2813.
E-mail addresses: gaozhw@lzu.edu.cn (Z.-W. Gao), KYL2813@yonsei.ac.kr (K.Y. Lee).

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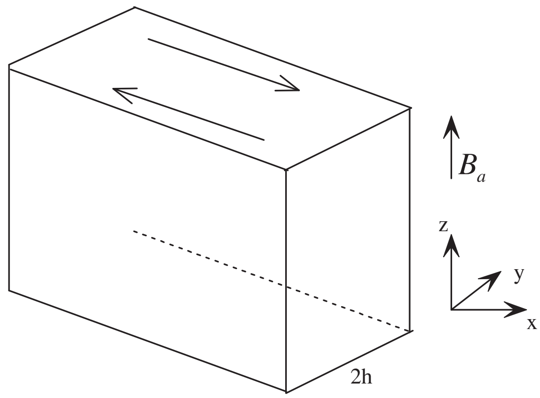

Fig. 1. Sketch of rectangular slab superconductor place in a magnetic field B_a.

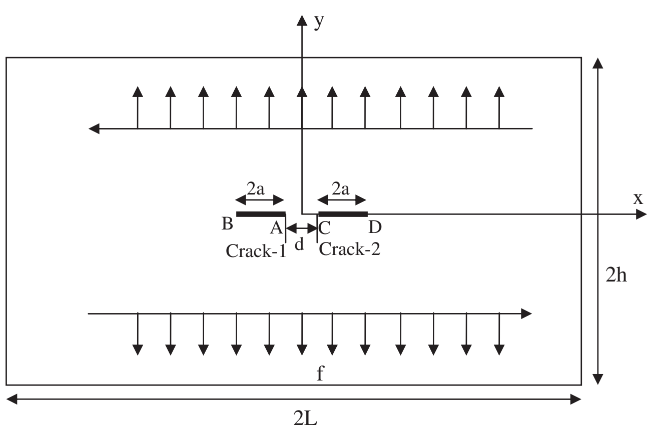

Fig. 2. Rectangular slab superconductor containing two collinear cracks.

the fracture problem based on the following assumptions: (1) the demagnetization effects are neglected; (2) the magnetic behavior can be described by the critical-state Bean model [17]; (3) the superconductor is assumed to be linear elastic.

According to the critical-state Bean model [17], the electromagnetic force arising from flux pinning is body force [5]. The body forces have been given in the form

$$f_y = -\frac{1}{2\mu_0}\frac{\partial}{\partial y}B(y)^2 \tag{1}$$

We will now investigate plane strain problem for superconductor material.

Due to the symmetry of the problem, half of the plate is modeled, with symmetry along the x-axis. The boundary conditions are:

$$u = 0, \quad v = 0 \quad \text{for} \quad x = -L \tag{2}$$

$$u = 0, \quad v = 0 \quad \text{for} \quad x = L \tag{3}$$

where u and v are the displacement in the x- and y-direction, respectively.

Finite element analysis is a powerful technique for computational analysis of structures. Fracture analysis of the plane strain problem of the superconductor crack shown in Fig. 2 is carried out by finite element method. The J-integral [18] is used for the fracture mechanics analysis of the specimen. The J-integral is defined in terms of the energy release rate associated with crack growth. In the context of quasi-static analysis the J-integral is defined in two dimensions as

$$J = \int_{\Gamma}\left(W\mathbf{I} - \mathbf{T}\frac{d\mathbf{u}}{\partial x}\right)d\Gamma \tag{4}$$

where Γ is any path beginning at the bottom crack surface and ending at the top surface; $\mathbf{T}$ is the traction given as $\mathbf{T} = \boldsymbol{\sigma}\cdot\mathbf{n}$, and $\mathbf{n}$ is the outward unit normal to Γ, $\boldsymbol{\sigma}$ is the stress tensor. For superconductor material W is the elastic strain energy density.

Elementary considerations can be used to show that for homogeneous materials J-integral is path independent and that J-integral is equal to G, the crack tip energy release rate on propagation of the crack. The calculation of J-integral is carried out after the finite element solution is obtained. Following Shih et al. [19], Eq. (4) can be expressed by the form

$$J = -\oint_{C+C_+ +\Gamma+C_-}\mathbf{m}\cdot\mathbf{H}\cdot\bar{\mathbf{q}}d\Gamma - \int_{C_+ +C_-}\mathbf{t}\cdot\frac{\partial\mathbf{u}}{\partial\mathbf{x}}\cdot\bar{\mathbf{q}}d\Gamma \tag{5}$$

where $\mathbf{H} = W\mathbf{I} - \boldsymbol{\sigma}\cdot\frac{\partial\boldsymbol{u}}{\partial x}$, $\bar{\mathbf{q}}$ is a sufficiently smooth weighting function within the region enclosed by the closed contour $C + C_+ + \Gamma + C_-$ and

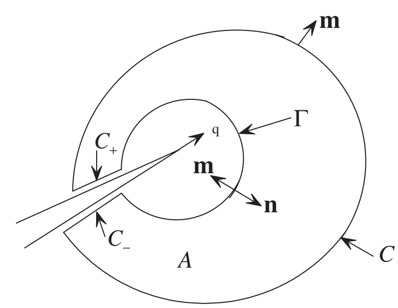

Fig. 3. Closed contour $C + C_+ + \Gamma + C_-$ encloses a domain A that includes the crack tip region as $\Gamma \to 0$.

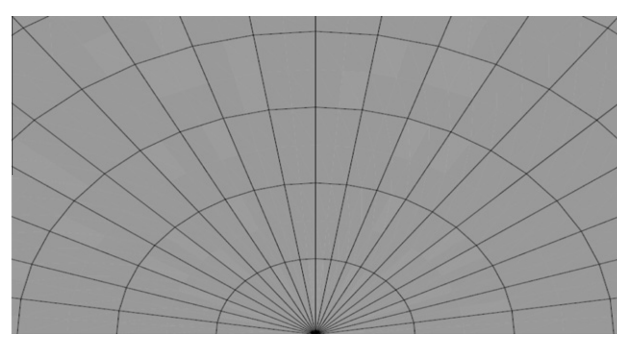

Fig. 4. The mesh around the crack tip.

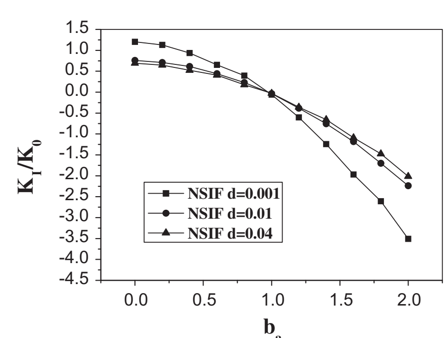

Fig. 5. The normalized stress intensity factors of crack tip A and C as the applied field are decreased from $b_{a,\max} = 2$ to $b_a = 0$. The NSIF increase when the distance between the two collinear cracks decreases.