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Simulation study for the orientation of the driven vortex lattice in an amorphous superconductor

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A B S T R A C T

We investigate the orientation of the vortex lattice driven by an applied current by means of numerical simulations based on the time-dependent Ginzburg–Landau (TDGL) theory. A lattice order is restored by a current driving of vortices under the influence of random vortex pinnings. The orientation of the moving vortex lattice is different between the presence and the absence of vortex pinnings. We show results of TDGL simulations for these phenomena.

1. Introduction

Much attention has been focused on vortices in type-II superconductors under magnetic fields. The vortices are driven into motion by applying currents. The moving vortices induce the electric field, and therefore the response of vortices to the applied current is detected by measuring the electric resistivity. The vortex pinnings by defects in materials affect the vortex motion and consequently influence the resistivity. In this way, the vortex and vortex pinning play an important role for electromagnetic properties of superconductors.

The vortices tend to form a lattice because of the repulsive interaction between them. It is known that the vortex pinnings disturb the lattice order in a static state. On the other hand, it has been discussed that a lattice order is restored by a current driving of vortices [1–4]. The dynamical lattice formation and deformation can be studied in a controlled way by applying currents, which stimulates broad interests in a wide range of science.

Recently, an ordered motion of vortices with small velocity was observed in a clean superconductor by the scanning tunneling microscopy [5,6]. A vortex lattice flow was investigated in an amorphous superconductor by the mode-locking resonance technique [7,8]. These experiments revealed that the orientation of the moving vortex lattice depends on the temperature and applied magnetic field. The lattice vector is perpendicular (parallel) to the direction of vortex motion in an intermediate-field region (in low- and high-field regions) in an amorphous superconductor [8]. Here, the lattice vector indicates the direction of the nearest neighbor vortex. As discussed later, it is expected that the lattice vector tends to be parallel to the direction of vortex motion in the absence of vortex pinnings. Therefore, the above experimental observation in an amorphous superconductor implies that the vortex pinning clearly influences the orientation of the moving vortex lattice.

By a molecular-dynamics (MD) simulation and an analytic study, it was indeed predicted that the lattice vector is perpendicular to the direction of vortex motion in the presence of vortex pinnings [9,10]. However, the magnetic-field dependence of the lattice orientation has not been understood yet. The experiments show that the vortices seem to move along the lattice vector near the upper critical field. Since the problem of the field dependence has still remained unsolved, further theoretical studies are required. Thus, simulation studies based on the time-dependent Ginzburg–Landau (TDGL) theory are expected to give a conclusive answer to the problem.

As a first step, we have performed the TDGL simulations at an intermediate magnetic field. In this paper, we report that the lattice vector certainly points to the direction perpendicular to the vortex motion under the influence of a random distribution of vortex pinnings, which is consistent with the experimental observation. This is the first TDGL simulation aiming to address the above issue.

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2. Ginzburg–Landau and Maxwell equations

To obtain the time development of the superconducting order parameter Δ and vector potential $\boldsymbol{A}$, we numerically solve the TDGL equation coupled with the Maxwell one. Those equations are written in a dimensionless form as

$$\frac{\partial(\Delta/\Delta_0)}{\partial(t/t_0)} = -\frac{1}{12}\left[\left\{\left|\frac{\Delta}{\Delta_0}\right|^2 - \left(1-\frac{T}{T_c}\right)\right\}\frac{\Delta}{\Delta_0} + \left(\frac{1}{\mathrm{i}}\frac{\partial}{\partial \boldsymbol{r}/\xi_0} - \frac{\boldsymbol{A}}{A_0}\right)^2 \frac{\Delta}{\Delta_0}\right], \tag{1}$$

$$\frac{\partial(\boldsymbol{A}/A_0)}{\partial(t/t_0)} = \left[\frac{\Delta}{\Delta_0}\left(-\frac{1}{\mathrm{i}}\frac{\partial}{\partial \boldsymbol{r}/\xi_0} - \frac{\boldsymbol{A}}{A_0}\right)\frac{\Delta^*}{\Delta_0} + \frac{\Delta^*}{\Delta_0}\left(\frac{1}{\mathrm{i}}\frac{\partial}{\partial \boldsymbol{r}/\xi_0} - \frac{\boldsymbol{A}}{A_0}\right)\frac{\Delta}{\Delta_0}\right] - 2\kappa^2 \frac{\partial}{\partial \boldsymbol{r}/\xi_0} \times \frac{\boldsymbol{H}}{H_0}. \tag{2}$$

Eq. (2) means $-\boldsymbol{j}_n = \boldsymbol{j}_s - \boldsymbol{j}_t$ with the normal current $\boldsymbol{j}_n$, supercurrent $\boldsymbol{j}_s$ and total current $\boldsymbol{j}_t$, where $-\boldsymbol{j}_n = -\sigma\boldsymbol{E} = \sigma\partial\boldsymbol{A}/\partial t$. The scalar potential is set equal to zero. In Eqs. (1) and (2) T_c is the transition temperature and κ is the Ginzburg–Landau (GL) parameter. We shall introduce the local suppression of $T_c(\boldsymbol{r})$, which acts as vortex pinning. The order parameter Δ, time t, vector potential $\boldsymbol{A}$, and magnetic field $\boldsymbol{H}$ are normalized by $\Delta_0, t_0 = 8\pi\kappa^2\xi_0^2\sigma/c^2, A_0 = \phi_0/(2\pi\xi_0)$, and $H_0 = \phi_0/(2\pi\xi_0^2)$, respectively. We have defined the order parameter Δ_0 and coherence length ξ_0 at zero temperature, the normal-state longitudinal conductance σ, light velocity c, and flux quantum ϕ_0.

We consider a two dimensional system in the xy plane. The magnetic field H_a is applied perpendicular to the plane. We discretize the system into a grid and use the link variable $U_\mu^{ij} = \exp[-\mathrm{i}\int_{r_i}^{r_j}(A_\mu/A_0)d\mu/\xi_0]$, when solving the TDGL and Maxwell equations [11–13]. Here, μ stands for x or y. The gauge-invariant differential terms are then replaced as

$$\left(\frac{1}{\mathrm{i}}\frac{\partial}{\partial r_\mu/\xi_0} - \frac{A_\mu}{A_0}\right)\frac{\Delta}{\Delta_0} \to \frac{1}{\mathrm{i}}\frac{U_\mu^{ij}\Delta_j/\Delta_0 - \Delta_i/\Delta_0}{a_\mu},$$

$$\left(\frac{1}{\mathrm{i}}\frac{\partial}{\partial r_\mu/\xi_0} - \frac{A_\mu}{A_0}\right)^2\frac{\Delta}{\Delta_0} \to -\frac{U_\mu^{ij}\Delta_i/\Delta_0 + U_\mu^{jk}\Delta_k/\Delta_0 - 2\Delta_j/\Delta_0}{a_\mu^2}$$

with the step size $a_\mu(\mu = x, y)$ and the sequential positions i, j, k along the μ coordinate on the grid. The dimensions of a unit cell of the grid are $a_x \times a_y$. The magnetic field $\boldsymbol{H}$ is obtained by the Stokes' theorem, $\exp[-\mathrm{i}\int_S(\boldsymbol{H}/H_0)\cdot\boldsymbol{n}dS/\xi_0^2] = \exp[-\mathrm{i}\int_c(\boldsymbol{A}/A_0)\cdot d\boldsymbol{l}/\xi_0]$, which can be calculated by using the product of link variables.

In our simulations, the size of the system is $L_x \times L_y = 50\xi_0 \times 200\xi_0$ with the grid unit $\xi_0 \times \xi_0$. The external current is applied in the x direction, and a periodic boundary condition is imposed in this direction. The system edges perpendicular to the y direction are considered as interfaces between a superconductor and a normal metal, near which T_c is set to be suppressed as shown in Fig. 1a. This T_c suppression along the interface assists the vortex entrance into the system. Thus, the interface does not significantly affect the dynamical lattice formation. The bulk transition temperature is denoted by T_{c0}. When investigating the influence of vortex pinnings, we introduce vortex-pinning sites, each of which is a point defect where T_c is locally suppressed within the range $0.9 \leqslant T_c/T_{c0} \leqslant 1$. The positions of pinning sites and the degree of T_c suppression are distributed randomly as shown in Fig. 1b. The number of pinning sites is 138 in the system $50\xi_0 \times 200\xi_0$.

The temperature T, applied magnetic field H_a, applied current density j_x, and GL parameter κ are set $T/T_{c0} = 0.5, H_a/H_0 = 0.2$, $j_x/j_0 = 3 \times 10^{-4}$ with $j_0 = \phi_0/(2\pi\xi_0^3)$, and $\kappa = 3$, respectively. The applied current in the x direction generates a magnetic-field gradient in the y direction. Therefore, at the two boundary edges perpendicular to the y axis, we set the external fields at the values consistent with the applied current and the applied magnetic field. Δ is set zero at these edges. The minimal time interval is set $0.01t_0$ when solving the TDGL and Maxwell equations. We continue to solve the time development of these equations until an ordered lattice structure is stabilized after a steady vortex motion is attained.

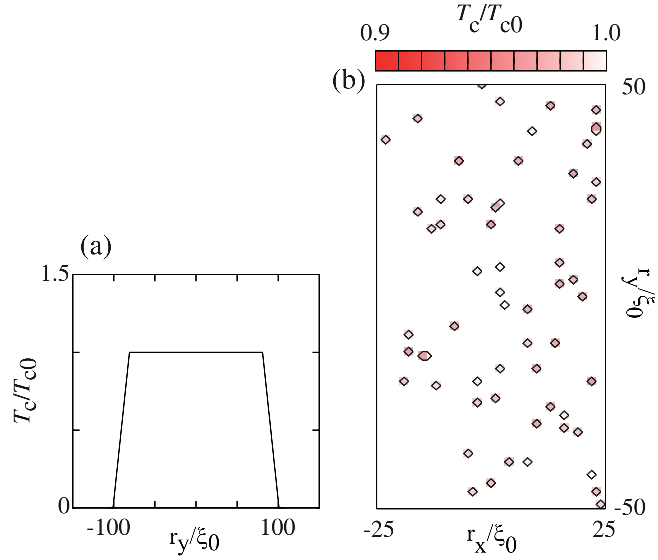

Fig. 1. (a) Spatical profile of the transition temperature $T_c(\boldsymbol{r})$ normalized by the bulk transition temperature T_{c0}. (b) Distribution of random vortex pinnings used in the present simulations. The region of $50\xi_0 \times 100\xi_0$ is displayed, which is a part of the system $50\xi_0 \times 200\xi_0$.

3. Results

We first consider a system without random vortex pinnings. A snapshot of moving vortices at a certain moment is shown in Fig. 2a, where the circles represent the positions of vortices. The vortices move from the top to the bottom in the negative y-direction. Another snapshot after the time interval $t/t_0 = 1000$ is presented in Fig. 2b. Then, the lattice structure of moving vortices is found to be already stabilized. We confirm that this lattice structure is unchanged permanently. The simulation result also shows that the lattice vector is parallel to the direction of vortex motion as denoted schematically in the lower panel of Fig. 2. Moreover, we find that this orientation of the vortex lattice is not determined by an influence of the interface perpendicular to the y axis. When the applied current is weak, it is expected that an interface effect appears and the vortices tend to align parallel to the interface along which T_c is suppressed as shown in Fig. 1a. However, the present result indicates that the vortices align perpendicular to the interface. Therefore, we conclude that, in the absence of random vortex pinnings, the moving vortex lattice is always oriented so that the lattice vector is parallel to the direction of motion.

On the other hand, when the applied current becomes substantially large, the moving lattice is destroyed by spontaneous nucleation of dislocations. A large field gradient due to a large applied current results in a significant spatial gradient of the density of vortices, by which dislocations of the vortex lattice are generated. Our result concerning the dislocations nucleation is consistent with the previous simulation study [4].

Next, we consider a vortex lattice flow under the influence of random vortex pinnings. The collective flow is simulated in this paper, while the plastic flow has been studied by another TDGL