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Control and synchronization of chaos in RCL-shunted Josephson junction using backstepping design

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Abstract

This paper investigates the control and synchronization of chaotic dynamics in RCL-shunted Josephson junctions based on backstepping nonlinear control theory. The method consists of a recursive approach that interlaces the choice of a Lyapunov function with the control. The method was employed to eliminate the chaotic behavior exhibited by the RCL-shunted Josephson junctions as well as to achieve global asymptotic synchronization between a drive-response RCLSJ system with different system parameters. Numerical simulations have been employed to verify the effectiveness of the control scheme; while the closed loop systems with the control are perfectly modeled using SIMULINK block.

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1. Introduction

The dynamical behavior of Josephson junctions (JJ) have for long attracted considerable research attention since Belykh, Pedersen and Soerensen published their work on chaos in Josephson junctions [1,2]. Thereafter, in 1980, Huberman et al. [3] presented numerical studies on chaos in JJ. Different models have been introduced to represent the JJ [4]. Amongst them are the Shunted linear resistive–capacitive junction (RCSJ) [4], the Shunted nonlinear resistive–capacitive junction (SNRCJ) [5], Shunted nonlinear resistive–capacitive–inductive junction (RCLSJ) [5–8] and the periodically modulated Josephson junction (PMJJ) [9,10]. The first two models of the Josephson junction contain two state variables and exhibit chaotic behavior with external sinusoidal signal, while the RCLSJ model which has been found to be very useful for high-frequency applications generate chaotic oscillations with external dc bias only. Wu and Li [10] recently carried out analytic and numerical investigations of the dynamics of periodically modulated Josephson junction (PMJJ) and showed that the PMJJ exhibits chaotic motion through the period-doubling cascade, when the amplitude of the modulation term is increased.

Beside the dynamics of a single Josephson junction, the dynamics of coupled Josephson junctions have also attracted research interest in the recent times [11–14]. For instance, intermittent synchronization has been reported in a resistively coupled chaotic JJ by Blackburn et al. [11]; while Dana et al. [13], recently investigated the synchronization behavior of uni-directionally coupled RCLSJ by means of a negative pulse forcing and observed intermittent synchronization. The robustness of the synchronization scheme to white noise was also established. A more recent study by Wang et al. [12], revealed a transition from synchronized state to quenching state in a mutually cou-

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pled JJ, with the two states co-existing for some coupling strength [12]. Indeed synchronization of chaotic systems can be interpreted in terms of the observer problem in control theory; and recent synchronization techniques address the problem of chaos synchronization based on control theory point of view; thus unifying the study of chaos control and chaos synchronization. In this direction, Ucar et al. [14] in a very recent paper extended the study of the dynamics of coupled Josephson junction by setting up the synchronization scheme for two-coupled RCLSJ in a master–slave configuration through suitably designed active controls.

The control and synchronization of chaotic systems have received increased research attention [15,16], since the classical work on chaos control was first presented by Ott et al. [17] in 1990, followed by the Pyragas time-delayed auto-synchronization control scheme [18]; and the pioneering work on the synchronization of identical chaotic systems evolving from different initial conditions was first introduced by Pecora and Carroll [19], the same year. The enormous research activities arising from the possible applications of chaos control and synchronization have motivated researchers to seek for various effective methods to achieve these goals. During the last one decade, the active control, which was originally proposed by Bai and Lonngren [20] has been extensively explored (For some recent applications, see for example Refs. [14,21–24]). In another development, backstepping design has been employed for controlling, tracking and synchronizing chaotic systems (see for example Refs. [25–29]); this is because backstepping design can guarantee global stability, tracking and transient performance for a broad class of strict-feedback nonlinear systems [30]. The technique is a systematic design approach and consists in a recursive procedure that skillfully interlaces the choice of a Lyapunov function and the control.

In a recent paper [31], we employed the backstepping approach to control intermittent chaotic transport in inertia ratchet that model the motion of a particle in an asymmetric periodic potential. In addition, we explored the property of backstepping scheme and proposed a simple active-backstepping approach for synchronizing different trajectories arising from different initial conditions in the ratchet model. In the present paper, we extend our investigation on backstepping control to RCLSJ model and present an active-backstepping scheme for the synchronization of two-coupled RCLSJ model, each subsystem evolving from different initial conditions and with different system parameters. The rest of the paper is organized as follows: In the next section, we describe briefly the RCLSJ model and present the backstepping designs in Section 3, together with numerical simulations. Section 4 concludes the paper.

2. The RCLSJ model

The RCLSJ model of JJ is described by the following set of first order differential equations:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \frac{1}{\beta_C}[i - g(y)y - \sin(x) - z], \\ \dot{z} &= \frac{1}{\beta_L}(y - z), \end{aligned} \tag{1}$$

where the nonlinear damping function $g(y)$ is approximated by a current–voltage relation between the two junctions and is defined by

$$g(y) = \begin{cases} 0.366 & \text{if } |y| > 2.9, \\ 0.061 & \text{if } |y| \leqslant 2.9 \end{cases} \tag{2}$$

x, y, and z represent the phase difference, the voltage at the junction, and the inductive current, respectively. β_C and β_L are constants that represent capacitive and inductive values, respectively. i is the dc external current. This dissipative model has been shown to have an attractor in a bounded region. For instance, when the parameters are set as follows: $\beta_C = 2.6$ and $\beta_L = 0.707$ for the initial conditions: $(x(0), y(0), z(0)) = (0, 0, 0)$, the RCLSJ model exhibits chaotic dynamics for the dc external current in the region $1 < i < 1.3$ [5,7–9,13]. For the numerical results the system (1) and (2) is modeled using Matlab/SIMULINK block, Fig. 1. In Fig. 2a, we show a chaotic attractor for $i = 1.15$, while in Fig. 2b, we display a periodic attractor for $i = 1.5$. Our objective here is to design control law based on recursive backstepping approach that will eliminate the chaotic behavior and drive the system to a stable equilibrium point. Secondly, we would extend our previous investigation of synchronization behaviors of this system by means of a new active-backstepping approach, which we proposed recently [31].

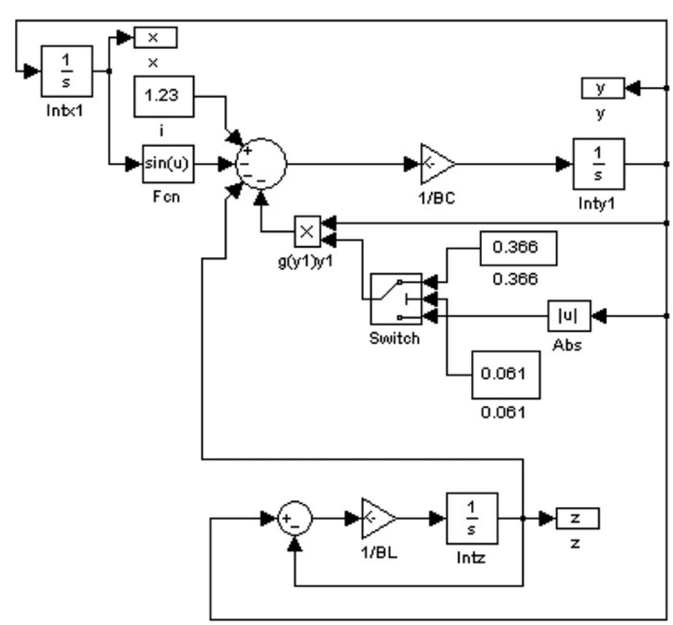

Fig. 1. SIMULINK block of the system (1) and (2).