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Current-driven dynamics in Josephson junction networks with an asymmetric potential

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ABSTRACT

Current-driven dynamics of Josephson junction networks (JJNs) is studied using numerical simulations. We consider a JJN with an asymmetric and periodic potential of vortices, which is realized by saw-tooth modulation of junction critical currents. When external ac currents are applied to the JJN in a magnetic field, there appears a ratchet effect, and then directed motion of vortices is induced in certain system parameter regimes. A ratchet behavior is observed even for JJNs with weak structural disorder. We clarify the vortex pinning and dynamics in the JJN as a ratchet system.

1. Introduction

Ratchet phenomena have been observed universally in diverse physical systems. In general, asymmetry existing in the system or in a driving force is relevant to the occurrence of the directed motion as a ratchet [1].

In recent years, as a promising new ratchet system, Josephson junction networks (JJNs) [2,3] have been studied using both experimental and theoretical methods [4,5]. Since JJNs are a well-controllable physical system, the strength of junction critical currents can be modulated spatially so that an asymmetric potential of vortices is created [4].

In this study, by numerical simulations of the RSJ model, we investigate ratchet effects of two-dimensional ac-current-driven JJNs with an asymmetric potential in the presence of a magnetic field. In the presence of structural disorder concerning positions of superconducting grains, the effects of disorder on the ratchet properties of JJNs are also clarified from the analyses of the current–voltage characteristics and the critical current.

2. Model of a Josephson junction network as a ratchet system

Fig. 1 shows a schematic sketch of a JJN considered here. We assume here that the JJN consists of a two-dimensional array of superconducting grains where an $N(=N_x \times N_y)$ square lattice structure is assumed, and each pair of the nearest-neighbor sites are connected by a Josephson junction in both the x- and y-direction. The external bias currents are injected (taken out) in the y-direction at the top (bottom) row. A magnetic field B is applied in the z-direction. In order to analyze the time evolution of superconducting phases of the JJN, we employ a current-driven RSJ model. The phase on the i-th site is denoted by ϕ_i. The equations of motion of ϕ_i are given by a set of N-coupled non-linear differential equations [6]:

$$\sum_{j}^{\mathrm{N.N.}} g_{ij}[\dot{\phi}_i - \dot{\phi}_j] = \frac{2e}{\hbar}\left[I_i - \sum_{j}^{\mathrm{N.N.}} J_{ij}\sin(\phi_i - \phi_j - A_{ij})\right], \tag{1}$$

where J_{ij} is the critical current of the junction between the i and j sites, $\sum^{\mathrm{N.N.}}$ means summation on the nearest-neighbor sites, A_{ij} the line integral of the vector potential given by $(2\pi/\Phi_0)\int \mathbf{A}\cdot d\mathbf{l}$, and $g_{ij} = 1/R_{ij}$ where R_{ij} is the junction resistance. $I_i = I$ (or $-I$) is the external bias current at the top (or bottom) of the array, and $I_i = 0$ otherwise. The external ac current, $I = I_{\mathrm{ac}}\sin(\frac{2\pi}{T}t)$, is applied to JJNs along the y-direction.

For the benefit of a later analysis, as shown in Fig. 1, we assume here that J_{ij} has an anisotropy which is expressed as $J_{ij} = J_x$ and J_y along the x- and y-directions, respectively [6]. To realize a ratchet potential, J_x is modulated only along the x-direction [5]. The modulated J_x is both periodic and asymmetric along the x-direction, and depends on n_x of the sites $n = (n_x, n_y)$ where $n_x(n_y)$ is an integer in $1 \sim N_x(N_y)$. In most JJNs considered in this study, $J_x(n_x)$ is given by the following saw-tooth form with a period N_{p},

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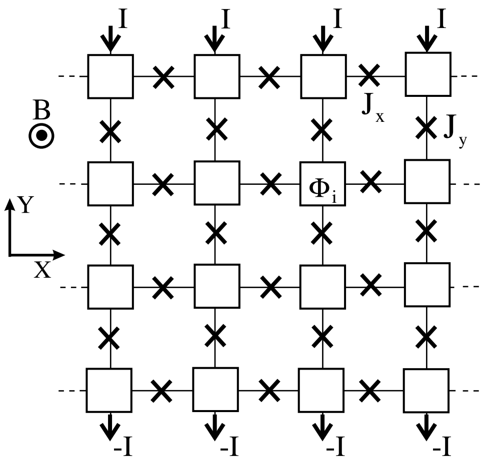

Fig. 1. Schematic sketch of a Josephson junction network.

$$J_x(n_x) = \begin{cases} J_x^{\min} + \frac{\Delta J_x}{N_p - 1}(n_x - 1), & \text{if } 1 \leqslant n_x \leqslant N_p, \\ J_x(n_x - N_p), & \text{if } n_x > N_p, \end{cases} \tag{2}$$

where $J_x^{\max}(J_x^{\min})$ is the maximum (minimum) value of $J_x(n)$, and $\Delta J_x = J_x^{\max} - J_x^{\min}$ is the modulation amplitude of $J_x(n)$ which corresponds to the amplitude of the ratchet potential. The average of $J_x(n_x)$ over the sites is given by J_x^0, $\langle J_x(n_x)\rangle_{n_x} = J_x^0$. The modulation of the junction critical currents creates the asymmetric ratchet potential of vortices with a period N_p. The ratchet potential represented in Eq. (2) is shown in Fig. 2.

We also consider JJNs with positional disorder of superconducting islands. The effect of disorder in JJN under a magnetic field is taken into consideration by using the Landau gauge of the vector potential. The Landau gauge is given by $\mathbf{A} = -By\hat{\mathbf{x}}$, and the magnetic field is $B = f\Phi_0/a^2$, where f is the magnetic frustration parameter and a the lattice spacing in the absence of positional disorder. For a disorder-free case, we can set $A_{i,i+1} = 2\pi f$ in the x-direction in the upper row and the other terms are given by $A'_{i,j} = A_i = 0$. According to Ref. [7], positional disorder of superconducting islands is taken into consideration by introducing displacement fields δ. Using δ, the position of the ith site of the JJN is given by $\mathbf{r}_i/a = (n^x + \delta_i^x, n^y + \delta_i^y)$, where $\mathbf{r}_i$ is the coordinate of the ith site of the ladder, and $\delta_i^x(\delta_i^y)$ is the deviation of the ith site and given by a random number distributed in $[-\delta, \delta]$. Then $A_{i,j}$, $A'_{i,j}$ and A_i have non-zero different values for each integral because of the random variables δ_i^x and δ_i^y.

In the numerical simulation, we assume periodic and free boundary conditions in the x- and y-directions, respectively. Numerical integrations of Eq. (1) are performed using a Runge–Kutta formula for parameters as $N_x = 16$, $N_y = 17$, $2e/\hbar = 1$, $R = 1$, $J_x^0 = 1$. The time step in the numerical integration is $dt = 0.1$. The time period of the ac currents is T/dt = 10,000. The spatial period of $J_x(n_x)$, which corresponds to the period of the ratchet potential, is $N_p = 4$. The strength of magnetic field is set as $f = 5 + \frac{1}{16}$. Note that, assuming $f = n + p/q$, the strength of disorder increases with n [8]. Hence, we use a finite n to investigate the effect of disorder, and we consider here a case of $p/q = 1/16$ as a low density of vortices.

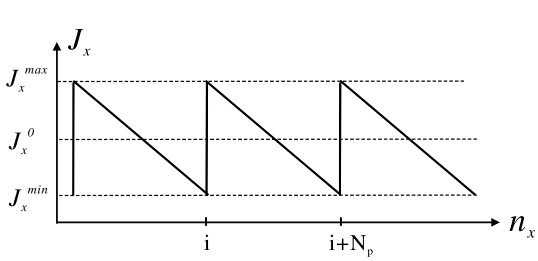

Fig. 2. Ratchet potential represented in Eq. (2).

3. Current–voltage characteristics and ratchet behavior

We calculate the time averaged voltage drops across the JJN in the y-direction in the presence of external ac currents,

$$V = \overline{\dot{\phi}_{\text{top}} - \dot{\phi}_{\text{bottom}}}, \tag{3}$$

where ϕ_{top} (ϕ_{bottom}) means the voltage averaged spatially at the top (bottom) row of arrays, and the overline means temporal average.

Fig. 3 shows the current–voltage ($I_{ac} - V$) characteristics obtained for $\delta = 0$. The curve (a) shows the $I_{ac} - V$ characteristic for $J_x(n_x)$ given by Eq. (2), and the curve (b) for the $J_x(n_x)$ with a saw-tooth form oriented in the direction opposite to that for Eq. (2). For both curves, there exist finite critical currents I_c, below which $V = 0$, and the strengths of I_c are almost the same. The sign of voltage V is different the two curves (a) and (b), and an inversion symmetry of $V \leftrightarrow -V$ is confirmed approximately. This shows voltage rectification using asymmetric potentials. These behaviors reflect the fact that a directed motion (flow) of vortices are realized, and the flow direction of vortices for (a) is opposite to that for (b). Therefore, it is obvious that a ratchet effect of vortices works in the JJN. The direction of vortex motion is determined by the asymmetric form of the ratchet potential which reflects the modulation form of $J_x(n_x)$.

Each curve in Fig. 3 has a large peak and a long tail in a large I_{ac} regime. Similar peak structure is observed generally also in other ratchet systems [9,10]. The complicated behavior of the long tail part of the $I_{ac} - V$ curves is attributed to the creation of vortex–antivortex pairs excited by strong driving due to large ac currents [5].

In Fig. 4, the $I_{ac} - V$ characteristics are plotted for different values of ratchet amplitude ΔJ_x. There exists a large distinct peak for each curve at a small I_{ac} just above I_c. The heights of the peaks increase with ΔJ_x. The critical currents, above which directed ratchet flow appears, also increase linearly with ΔJ_x. This means that, as ΔJ_x is increased, the vortex pinning force due to the ratchet potential is enhanced. However, when once the depinning occurs, the directed motion of vortices is strengthened in the ratchet potential with a steep slope due to large ΔJ_x.

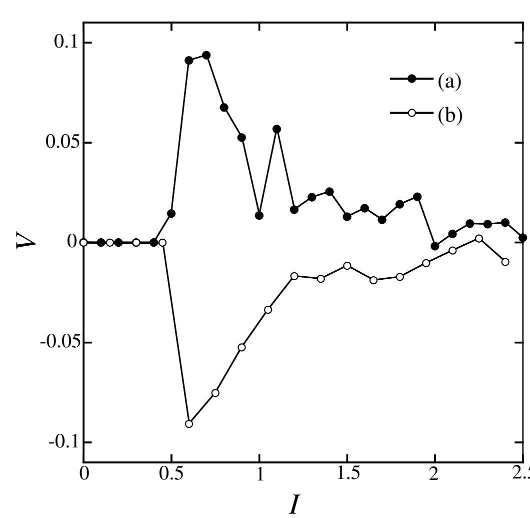

Fig. 3. Current–voltage characteristics for two kinds of asymmetric ratchet potentials.