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On the interpretation of muon-spin-rotation experiments in the mixed state of type-II superconductors

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Abstract

We argue that claims about magnetic field dependence of the magnetic field penetration depth λ, which were made on the basis of muon-spin-rotation (μSR) studies of some superconductors, originate from insufficient accuracy of theoretical models employed for the data analysis. We also reanalyse some of already published experimental data and demonstrate that numerical calculations of Brandt [E.H. Brandt, Phys. Rev. B 68 (2003) 54506] may serve as a reliable and powerful tool for the analysis of μSR data collected in experiments with conventional superconductors. Furthermore, one can use this approach in order to distinguish between conventional and unconventional superconductors. It is unfortunate that these calculations have practically never been employed for the analysis of μSR data.

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1. Introduction

Muon-spin-rotation (μSR) experiments in the mixed state of type-II superconductors provide unique information about superconducting properties of the investigated sample. An important advantage of this method is that muons probe the bulk of the sample and therefore, the results are not distorted by possible imperfections of the sample surface. At the same time, in order to extract quantitative results from μSR measurements, a detailed model of the magnetic field distribution in the mixed state is needed. As well as we are aware, only the Ginzburg–Landau (GL) theory [1] of the Abrikosov vortex lattice [2] is developed to such a level [3–5]. As was recently demonstrated, if an adequate model is available, not only the magnetic field penetration depth λ but also the upper critical field H_{c2} can be found from μSR data collected in different applied magnetic fields [6]. It has to be remembered, however, that theoretical calculations of Refs. [3–5] are related to superconductors with one and isotropic energy gap only. This is why, this kind of analysis should be used with extreme caution in the case of unconventional superconductors, in which the applicability of theoretical models is not obvious.

We also point out a very interesting and promising approach which was developed in Refs. [7–10]. In these works, a microscopic theory was used for calculation of the mixed state parameters. An important advantage of this approach is that the results are not limited to conventional superconductors and it can be used at temperatures well below T_c both for s- and d-wave pairing.

In recent years, μSR measurements were widely used for studying of different unconventional superconductors such as high-T_c materials, MgB_2 and others. Some very

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interesting results were obtained. It was demonstrated that in some cases the magnetic field penetration depth λ and the superconducting coherence length ξ, evaluated from μSR measurements, depend on the applied magnetic field (see, e.g. [11–19]). This result, however, contradicts the GL theory, which was used as a basis for the data analysis. This contradiction is a clear sign that the corresponding models are not adequate for describing the magnetic field distribution in the mixed state of these compounds and rises the question about physical meanings of $\lambda(H)$ and $\xi(H)$ obtained in such a way. As we argue below, magnetic field dependences of λ and ξ cannot be obtained from μSR experiments if the conventional GL theory or the London model were employed for the analysis of experimental data. Moreover, because in the mixed state the superconducting order parameter is not spatially uniform, there is no reasonable way to define either λ or ξ. In other words, the physical meanings of magnetic field dependences of λ and ξ, evaluated from μSR data, are quite different from traditional definitions of these two lengths. This circumstance was recognized in Refs. [20–22] where it was pointed out that $\lambda(H)$, evaluated in such a way, represents some fit-parameter rather than the magnetic field penetration depth. We underline that the same should also be addressed to $\xi(H)$ dependences. In the following section, in order to avoid confusion, we shall use λ_0 and ξ_0 to denote values λ and ξ for $H \to 0$.

2. Conventional superconductors

Superconductors with s-pairing and one energy gap, we shall consider as conventional, independent of their pairing mechanism. Because the GL theory is traditionally used for analyses of μSR data, we limit our consideration to this theory.

The magnetic field penetration depth λ_0 together with the zero-field coherence length ξ_0 represent two fundamental lengths of the GL theory. If their values for some particular temperature T are known, one can calculate the GL parameter

$$\kappa(T) = \lambda_0(T)/\xi_0(T), \tag{1}$$

the thermodynamic critical magnetic field

$$H_c(T) = \frac{\Phi_0}{2\sqrt{2}\pi\lambda_0(T)\xi_0(T)}, \tag{2}$$

the upper critical field

$$H_{c2}(T) = \sqrt{2}\kappa H_c(T) = \frac{\Phi_0}{2\pi\xi_0^2(T)}, \tag{3}$$

the lower critical field

$$H_{c1}(T) = \frac{\ln\kappa(T) + \alpha(\kappa)}{\sqrt{2}\kappa(T)} H_c(T) = [\ln\kappa(T) + \alpha(\kappa)]\frac{\Phi_0}{4\pi\lambda_0^2(T)} \tag{4}$$

with $\alpha(\kappa) = 0.49693 + \exp[-0.41477 - 0.775\ln\kappa - 0.1303(\ln\kappa)^2]$ [5]. Furthermore, in the case of conventional superconductors, any characteristics of the sample for any value of an applied magnetic field may also be calculated and expressed via λ_0 and ξ_0. Very detailed numerical calculations of different parameters of the mixed state for a very wide range of κ and for magnetic fields ranging from H_{c1} to H_{c2} are presented in Ref. [5].

Muons probe the distribution of the magnetic induction in the sample. In high-κ superconductors and low-magnetic inductions B, contributions of vortex cores can be neglected (London limit) and the distribution of the magnetic induction around a single vortex line may be written as

$$B(r) = \frac{\Phi_0}{2\pi\lambda_0^2}K_0(r/\lambda_0), \tag{5}$$

where r is the distance from the vortex center, Φ_0 is the magnetic flux quantum and K_0 is the modified Bessel function. Because Eq. (5) is obtained from the London theory, it gives an unphysical divergence of B at $r = 0$. In order to improve Eq. (5), an appropriate cutoff has to be introduced [23–25]. It should be remembered, however, that the results of Refs. [23–25] can be considered as sufficiently accurate in low-magnetic fields $H \ll H_{c2}$ only. If this condition is not satisfied, numerical solution of the GL equations must be used for a reliable analysis μSR data. The magnetic induction distribution may be calculated as a linear superposition of inductions created by different vortices (see, for instance, Ref. [25]).

By measuring muon relaxation rates, one obtains the distribution of the magnetic induction $P(B)$ experimentally, which allows to calculate the variance of the magnetic induction

$$\sigma^2 = \left(\overline{B^2(r) - \overline{B}^2}\right), \tag{6}$$

where $\overline{\cdots} = (1/V)\int\cdots d^3r$ means spatial averaging over superconductor of volume V. If the distribution of the magnetic induction around vortices is known, σ can also be calculated theoretically. According to [5]

$$\sigma = F(\kappa, B/B_{c2})/\lambda_0^2, \tag{7}$$

where the parameter F depends on κ and B/B_{c2}. If the value of F is known, λ_0 may straightforwardly be evaluated. In the case of $\kappa \gg 1$ and $b \ll 1$, $F \approx 0.061\Phi_0$. In other situations, reliable results can be obtained from Ref. [5]. Eq. (7) may also be written as $\sigma = (2\pi H_{c2}/\Phi_0)\ F(\kappa, B/B_{c2})/\kappa^2$. This representation may be convenient if evaluation of κ is preferable.

While the zero-field value of λ enters the theory, the actual magnetic field penetration depth is field dependent. According to the original Ginzburg and Landau publication [1], if the magnetic field is parallel to the sample surface,

$$\lambda(H) = \lambda_0[1 + f(\kappa)H/H_c]. \tag{8}$$