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0- and π-states in Josephson coupling through magnetic layers

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Abstract

We study the Josephson coupling energy, E_J, between two superconductors (SCs) through magnetic layers, i.e., ferromagnetic metal (FM) and antiferromagnetic insulator (AFI). By the tunneling Hamiltonian approach, analytical formulae of E_J are given in the fourth order perturbation theory as to the tunneling matrix element. In the former case, the E_J exhibits a damped oscillatory dependence on the thickness of the FM, and shows a transition between the 0- and the π-Josephson couplings. In the latter case, the magnetic exchange interaction in the AFI plane suppresses the π-Josephson coupling, and the 0-Josephson coupling leads to coexistence between SC and AFI. It is found that the origin of π-Josephson coupling in the SC/AFI/SC junction is different from that in the SC/FM/SC one.

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1. Introduction

Josephson effect is a quantum mechanical phenomenon, in which the current flows without voltage drop between two superconductors (SCs). In conventional Josephson junctions, a band insulator or a paramagnetic metal has been used to separate two superconductors (SCs) [1–4]. The current flows to minimize the coupling energy between two SCs, and is given by $I = 2e\partial F/\partial\varphi \equiv E_J \sin\varphi$, where F is the Free energy, and φ is a phase difference between two SCs. Hence, if the Josephson coupling energy, E_J, is positive, the minimum of the energy is obtained at $\varphi = 0$. It is called the 0-state.

In the SC/FM/SC junction with ferromagnetic metal (FM), on the other hand, the current–phase relation (CPR) is shifted by π from that in the conventional Josephson junctions [5–13]. It is called the π-state. This phase shift originates from the fact that E_J becomes negative depending on thickness of the FM layer, d_F, and temperature, T. It was shown that E_J exhibits a damped oscillatory behavior as a function of d_F, since Cooper pairs in the FM have a finite center-of-mass momentum proportional to the magnetic exchange splitting between an up- and a down spin bands [14–16]. However, the T-dependence of E_J is not still clear. Then, we formulate E_J in the SC/FM/SC junction by using a tunneling Hamiltonian in the fourth order perturbation theory. It is found that the magnetic fluctuation plays an important role in the 0–π transition, in particular, with T.

Another interesting case for the π-state is the SC/AFI/SC junction, which is realized in multilayered cuprates such as $HgBa_2Ca_4Cu_5O_y$ [17,18] and in a heterostructure composed of $La_{1.85}Sr_{0.15}CuO_4$ and La_2CuO_4 [19]. In these materials, a coexistence of superconducting (SC) and antiferromagnetic (AF) states has been observed, while the SC planes are separated by AF planes in the direction perpendicular to the planes. In general, a Josephson coupling between SC planes is necessary both to stabilize the bulk

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SC state and to enhance the SC critical temperature, T_{SC}, in layered superconductors. Therefore, in the above coexistence, the Josephson coupling through the AF planes is required. In this paper, we will briefly summarize the results on the SC/AFI/SC junction [20], and apply them to a ferromagnetic insulator (FI) case.

2. E_J in a SC/FM/SC junction

We consider the BCS mean-field Hamiltonian in the s-wave SCs, and ferromagnetically ordered magnetic moment in the FM. The total Hamiltonian, H, is given by

$$H = H_{SC} + H_{FM} + H_T, \tag{1}$$

$$H_{SC} = H_L + H_R, \tag{2}$$

$$H_L = \sum_{k_L,\sigma} \xi_{k_L} c^\dagger_{k_L,\sigma} c_{k_L,\sigma} + \sum_{k_L} \Delta e^{i\varphi_L} c^\dagger_{k_L,\uparrow} c_{-k_L,\downarrow} + \text{h.c.}, \tag{3}$$

$$\xi_{k_L} = \frac{1}{2m} k_L^2 - \mu, \tag{4}$$

$$H_R = (L \to R), \tag{5}$$

$$H_{FM} = H_0 + H_{loc} + H_{imp} + H_{mag}, \tag{6}$$

$$H_0 = \sum_{k_{FM},\sigma} c^\dagger_{k_{FM},\sigma} \left(\frac{1}{2m} k_{FM}^2 - \mu - \sigma h_{ex} \right) c_{k_{FM},\sigma}, \tag{7}$$

$$H_{loc} = J_\parallel \sum_{\langle i,j \rangle} S_i^z \cdot S_j^z, \tag{8}$$

$$H_{imp} = \frac{u}{N} \sum_{k_{FM},q,\sigma} c^\dagger_{k_{FM}+q,\sigma} c_{k_{FM},\sigma}, \tag{9}$$

$$H_{mag} = -\frac{J_H}{2\sqrt{N}} \sum_{k_{FM},q,\sigma} \sigma\, \delta S_q^z c^\dagger_{k_{FM}+q,\sigma} c_{k_{FM},\sigma}, \tag{10}$$

$$\delta S_q^z = S_q^z - \langle S_q^z \rangle, \tag{11}$$

$$H_T = H_T^L + H_T^R, \tag{12}$$

$$H_T^L = \sum_{k_L,k_{FM},\sigma} t e^{i(k_{FM}-k_L)r_L} c^\dagger_{k_L,\sigma} c_{k_{FM},\sigma} + \text{h.c.}, \tag{13}$$

$$H_T^R = \sum_{k_R,k_{FM},\sigma} t e^{i(k_{FM}-k_R)r_R} c^\dagger_{k_R,\sigma} c_{k_{FM},\sigma} + \text{h.c.} \tag{14}$$

In Eq. (2), $H_L(H_R)$ is the Hamiltonian of left (right) SC, where $\xi_{k_L}(\xi_{k_R})$ is the kinetic energy of electron with wavenumber $k_L(k_R)$ and μ is the chemical potential. The superconducting gap and the phase variable are denoted by Δ and $\varphi_L(\varphi_R)$, respectively. The annihilation and creation operators of electron with wavenumber vector k and spin σ are denoted by $c_{k,\sigma}$ and $c^\dagger_{k,\sigma}$, respectively. Eq. (7) describes the kinetic energy of electron in the FM with the magnetic exchange energy, $h_{ex} = (J_H/2)\langle S_q^z \rangle$, where $\langle S_q^z \rangle$ is determined by the self-consistent equation as, $\langle S_q^z \rangle = (1/2)\tan h(2T_{FM}/T\langle S_q^z \rangle)$. The Curie temperature, T_{FM}, is defined as $T_{FM} = 6J_\parallel$. J_H is a coupling constant between an electron and a localized moment in the FM with N sites, and S_q^z denotes the z component of the localized spin with wavenumber vector q. For the ferromagnetically ordered moment in Eq. (8), we adopted a mean-field Hamiltonian given by $J_\parallel \sum_{\langle i,j \rangle} S_i^z \cdot S_j^z \to 6J_\parallel \langle S^z \rangle \sum_i S_i^z$, where $\langle i,j \rangle$ indicates the sum of nearest neighbor sites. A non-magnetic impurity scattering in the FM is given by Eq. (9), where u is an impurity potential. We adopt Eq. (10) to describe the magnetic fluctuation in the FM. The tunneling Hamiltonian is given by H_T in Eq. (12), where $r_L(r_R)$ is the position vector of the interface between the left (right) SC and the FM.

The Josephson coupling energy, E_J, is calculated in the forth order perturbation theory as regards t. For h_{ex}/μ, $\omega_n/\mu \ll 1$, E_J in the clean system is given by

$$E_J R_0 = \Delta^2 \left(T \sum_{\omega_n} \frac{1}{\omega_n^2 + \Delta^2} e^{-2|\omega_n| d_F / v_F} \right) \cos\left(\frac{2h_{ex}}{v_F} d_F \right) \tag{15}$$

$$= \frac{1}{2\pi\Delta} \int_0^\infty dx \sin x \left[\frac{\pi T}{\Delta} \operatorname{cosech}\left(\frac{\pi T x}{\Delta} + \frac{d_F}{\xi_T} \right) \right] \times \cos\left(\frac{2h_{ex}}{v_F} d_F \right), \tag{16}$$

where $\xi_T = v_F/2\pi T$ and R_0 is a constant determined by the material and the interface. The Matsubara frequency and the Fermi velocity of electrons are denoted by ω_n and v_F, respectively. Eq. (16) exhibits the damped oscillatory behavior, whose period is determined only by h_{ex}. Hence, the 0–π transition occurs with d_F. Moreover, if h_{ex} is strongly changed with T, it may be possible to observe the 0–π transition with T even in the clean system.

On the other hand, in the dirty FM, the diffusive motion of electrons is included by a sequential sum of the non-magnetic impurity and the scattering by magnetic fluctuation. By taking the dirty limit of $\omega_n\tau$, $h_{ex}\tau$, $v_F|Q|\tau \ll 1$, E_J in the dirty FM is given by

$$E_J R_0 = \Delta^2 T \sum_{\omega_n > 0} \frac{1}{\omega_n^2 + \Delta^2} \exp\left(-\frac{d_F}{\xi_{F_+}} \right) \cos\left(\frac{d_F}{\xi_{F_-}} \right), \tag{17}$$

$$\xi_{F_+} \equiv \left[\frac{D}{\sqrt{(\omega_n + 1/\tau_s)^2 + h_{ex}^2} + (\omega_n + 1/\tau_s)} \right]^{1/2}, \tag{18}$$

$$\xi_{F_-} \equiv \left[\frac{D}{\sqrt{(\omega_n + 1/\tau_s)^2 + h_{ex}^2} - (\omega_n + 1/\tau_s)} \right]^{1/2}, \tag{19}$$

$$\frac{1}{\tau_s} = \left(\frac{J_H}{2} \right)^2 \frac{2\pi}{\mu} \langle \delta S^z \delta S^z \rangle, \tag{20}$$

where $D = \frac{1}{3} v_F^2 \tau$, $1/\tau \equiv (1/\tau_{imp} + 1/\tau_s)$, $1/\tau_{imp} = 2\pi N_F n_i u^2$. N_F is the density of states on the Fermi surface. The density of the non-magnetic impurity is denoted by n_i. In the mean-