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Complex dynamics of mode-locking in a Josephson junction network

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Abstract

Mode-locking phenomena in a disordered Josephson junction network driven by a periodic current pulse are studied using a numerical simulation. For a ladder type of the network structure with disorder, the dynamics of local voltages and the voltage–voltage correlation are investigated in a regime between the mode-locking and unlocking states. We clarify the relationship between locking–unlocking processes and the phase pinning due to disorder.

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Keywords: Josephson junction network; Mode-locking; Pinning

1. Introduction

Mode-locking phenomena have been observed universally in diverse dynamical systems. Among them, complicated mode-locking phenomena appear in systems with many degrees of freedom. As a well-controlled physical system which has many degrees of freedom, Josephson junction networks (JJNs) have been studied using both experimental and theoretical methods [1,2]. The JJNs consist of networks of superconducting grains coupled with Josephson junctions, For JJNs, mode-locking occurs when they are driven periodically by bias currents. There appear diverse mode-locking phenomena such as fractional giant Shapiro steps [3], and complicated problems concerning mode-locking remain unresolved for JJNs.

In this study, we investigate complex dynamics of mode-locking phenomena in a JJN. We consider here a simple JJN with a ladder type of the structure among several possible structures of JJNs. This type of JJN is called a Josephson junction ladder (JJL). If structural disorder in positions of grains exists in the presence of a magnetic field, the effect of the disorder causes pinning of phases or vortices. The strength of the pinning governs the strength of the critical current of the JJL. The relationship between such pinning effects and the mode-locking dynamics has not been clarified yet. In this paper, we investigate this issue and clarify the dynamics of locking–unlocking processes for both weak and strong pinning cases.

2. Model of a Josephson junction ladder

Fig. 1 shows a schematic sketch of the JJL considered in this study. We assume here that the JJL considered in this study has two rows parallel to the *x*-direction, and each row has N_x superconducting sites. Each pair of the nearest-neighbor sites is connected by a Josephson junction in both the *x*- and *y*-directions. The lattice constants of plaquettes of the ladder are the same in both the directions. Bias currents are injected (taken out) at the upper (lower) row along the *y*-direction. A magnetic field B is applied in the *z*-direction. We consider here the current-driven resistively shunted junction (RSJ) model of the JJL to analyze the time evolution of the phases of superconducting wave functions. The superconducting phases on the *i*th site of the upper and lower rows are denoted by ϕ_i and ϕ'_i, respectively.

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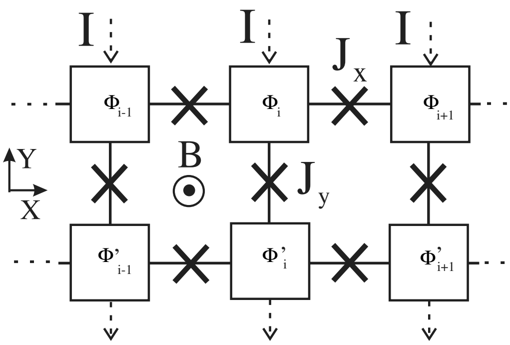

Fig. 1. Schematic sketch of a Josephson junction ladder.

In the present system, the bias current $I(t)$ is a periodic pulse which is given by

$$I(t) = \begin{cases} I & \text{if } 0 < t \leqslant t_{on}, \\ 0 & \text{if } t_{on} < t \leqslant t_{on} + t_{off} = T, \\ I(t-T) & \text{if } t > T, \end{cases} \quad (1)$$

where I is the strength of the pulse, and t_{on} and t_{off} are the duration and interval of the pulse, respectively.

In the presence of the bias currents, the equations of motion of the phases [4,5] are given by

$$\frac{\hbar}{2eR}[3\dot{\phi}_i - \dot{\phi}_{i-1} - \dot{\phi}_{i+1} - \dot{\phi}'_i] = I(t) + J_x[\sin(\phi_{i-1} - \phi_i - A_{i,i-1}) + \sin(\phi_{i+1} - \phi_i - A_{i,i+1})] + J_y \sin(\phi'_i - \phi_i - A_i), \quad (2)$$

and

$$\frac{\hbar}{2eR}[3\dot{\phi}'_i - \dot{\phi}'_{i-1} - \dot{\phi}'_{i+1} - \dot{\phi}_i] = -I(t) + J_x[\sin(\phi'_{i-1} - \phi'_i - A'_{i,i-1}) + \sin(\phi'_{i+1} - \phi'_i - A'_{i,i+1})] + J_y \sin(\phi_i - \phi'_i + A_i), \quad (3)$$

where $J_{x(y)}$ is the critical current of junctions in the $x(y)$-direction, and R the junction resistances. The line integrals of the vector potential $(2\pi/\Phi_0)\int_r^{r'} \mathbf{A} \cdot d\mathbf{l}$ are denoted by $A_{i,j}$ along the x-direction of the upper row, $A'_{i,j}$ along the x-direction of the lower row, and A_i along the y-direction between the upper and lower rows.

Here, we assume that the JJL has structural disorder, and consider a case of strong disorder. Effects of strong structural disorder under a magnetic field are taken into consideration by using a random gauge in the vector potential [4,5]. For this gauge, the line integrals of $\mathbf{A}$ along the y-direction, A_i, are considered as a random number distributed uniformly in $[-\pi, \pi]$, and all terms along the x-direction are set to be zero: $A_{i,j} = A'_{i,j} = 0$. This model is called the random gauge RSJ model. In this paper, we do not take an average on the random configuration of A_i because we confirmed that the results are essentially independent of the difference in the random configuration.

In the numerical simulation, we assume periodic and free boundary conditions in the x- and y-directions, respectively. Numerical integrations of Eqs. (2) and (3) are performed using a Runge–Kutta formula. The values of parameters are set as $N_x = 200$, $2e/\hbar = 1$, $R = 1$, and $J_x = 1$. The time step in the numerical integration is $dt = 0.03$. For the current pulse, $t_{on}/dt = 60$ and $t_{off}/dt = 200$, and hence $T/dt = 260$. The critical currents of junctions are important parameters, and $J_x = 1$ and $J_y = 1$ and 5 are employed.

3. Current–voltage characteristics

We calculate the time averaged voltage drops across the JJL in the y-direction along the bias current,

$$V = \frac{1}{N_x}\sum_i \langle \dot{\theta}_i \rangle_t, \quad (4)$$

where $\theta_i (= \phi_i - \phi'_i)$ is the phase difference between the two rows at site i, and $\langle\ \rangle_t$ means temporal average. If mode-locking exists in the JJL, voltage steps are observed in the I–V characteristics. There are fundamental mode-locking states, where the values of the voltage steps are given by $V_n^{ML} = n2\pi/T$, where n is an integer. In these states, the averaged phase difference $\frac{1}{N_x}\sum_i \theta_i$ advances by $2\pi n$ during the time period T. Another mode-locking condition is also possible, and then several locked steps will appear at certain values of the voltage in the I–V characteristics.

In Fig. 2, the I–V characteristics are plotted for $J_y = 1$ and 5, where V is normalized with the value of the $n = 1$ voltage step, $V_{n=1}^{ML}$. There appear some ML steps in the I–V curve. For both values of J_y, the voltages of most profound steps are given by those of the fundamental locking V_n^{ML}. Another step of the mode-locking is also observed, the normalized voltages of which are given by $V_{p,q}^{ML} = p/q$ where p and q are integers. These steps are narrow compared with the fundamental ones. The critical current for

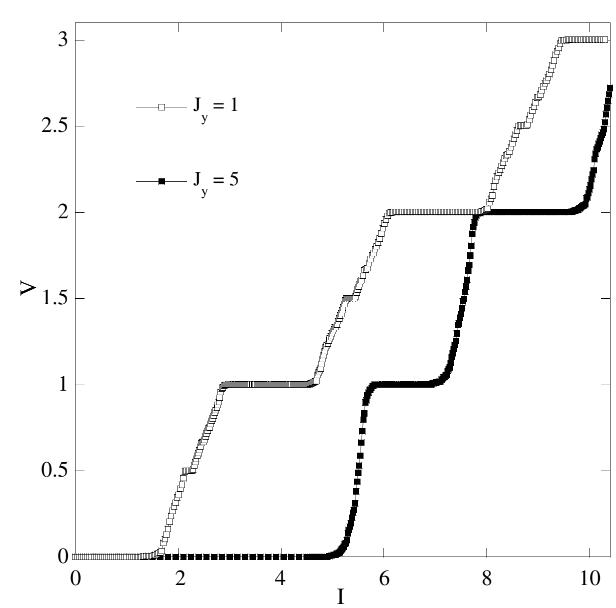

Fig. 2. Current–voltage characteristic.