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# Triplet vortex state in magnetic superconductors – Effects of boundaries

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**Abstract**

A mesoscopic superconductor with a magnetic moment in its center exhibits confined vortex loops with threefold symmetry in its center. Here, we show how the boundaries can affect this symmetry by considering a superconducting cube and sphere.


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Superconductivity and magnetism are mutually exclusive types of order known to coexist in the recently discovered ferromagnetic superconductors [1] and in the superconducting ferromagnets [2,3]. Nano-engineered superconductors with magnetic defects in its surface show coexistence between superconductivity and magnetism in the vortex pattern [4,5]. Recently, several nano-composites were also fabricated, such as $MgB_2$ with embedded magnetic $Fe_2O_3$ nanoparticles [6], or Gd particles incorporated in a Nb matrix [7]. We believe that such compounds can be used to obtain the vortex pattern predicted here, which is made of confined vortex loops (CVLs) and external vortex pairs (EVPs). In mesoscopic superconductors the volume to surface area ratio is small [8], and strong lateral confinement imposes the formation of giant vortices [9,10], while the shape of the boundary directs the symmetry of the final vortex configuration [11]. Similar effects are also observed here in our vortex patterns.

In this paper we consider the vortex pattern resulting from a static magnetic moment $\mu$ in the center of a finite size superconductor. A sub-micron superconducting particle has its center taken by a point like magnetic dipole whose magnetic field leads to CVLs. These loops eventually spring to the surface and give rise to vortex pairs, because the magnetic moment's north and south poles are the only source and sinkhole of vortices, respectively. Thus, as seen from the surface, a vortex and an anti-vortex are just the tips of a single EVP. The number of CVLs and of EVPs determines the vortex state and in the present study we restrict the magnetic moment strength to $\mu \geqslant \mu_0$, $\mu_0 \equiv \Phi_0\xi/2\pi$. We define the magnetic moment through a length, $d$, $\mu \equiv \Phi_0 5d/8\pi$, and thus, $\mu/\mu_0 = 5d/4\xi$. All the temperature dependence of the present system is in the ratio $\mu/\mu_0$. Recently, we showed [12] that CVLs arise in triplets from the $H_{c2}$ core surrounding the magnetic moment. The growth of a CVL beyond the superconductor boundaries will eventually transform it into an EVP state. With increasing $\mu$ the CVLs approach the external surface, which is expected to have some influence on the symmetry of the state. In this paper we find that the CVLs remain in a

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triplet state if far from the surface. for this purpose two different geometries are studied whose surfaces eventually affect the vortex pattern: a sphere with diameter 30 $\xi$, and a cube with side 30 $\xi$. In the latter case the magnetic moment is taken parallel to two of the cube's faces.

The vortex pattern, as a function of $\mu$, is obtained by numerically solving a gauge invariant discrete version of the Ginzburg–Landau (GL) theory on a cubic cell with side $36\xi$. The numerical minimization of the GL free energy is done through the method of simulated annealing [12] using a mesh grid of $N^3$ points ($N = 46$), containing the finite size superconductors inside. The Meissner shielding is not included, and so, Ampére's law is safely ignored. The present description is restricted to a hard type II superconductor whose free energy is given by,

$$f = \int \frac{dv}{V} \tau \xi^2 \mid (\mathbf{\nabla} - \frac{2\pi i}{\Phi_0}\mathbf{A})\psi|^2 - \tau \mid \psi|^2 + \frac{1}{2} \mid \psi|^4, \qquad (1)$$

expressed in units of the critical field energy density, $H_c^2/4\pi$. The magnetic field is $\boldsymbol{B} = \boldsymbol{curl A}$, $\boldsymbol{A} = \boldsymbol{\mu} \times \boldsymbol{r}/|\boldsymbol{r}|^3$, and we impose that in the center of the cubic cell the order parameter vanishes. Notice that (i) a non-superconducting core naturally evolves around the center and acquires radius size of the order of $\xi$; (ii) the magnetic field is continuous across the boundary; (iii) the superconducting current normal to the surface vanishes (deGennes' boundary condition). The non-linearity of the theory is fully treated in our procedure and this determines its ground and excited states. The shape of the finite superconductor enters directly into the free energy through a step-like function $\tau(\mathbf{x})$, equal to one inside the superconductor, and zero outside, as discussed in Ref. [12]. Fig. 1 shows the free energy and the induced magnetic moment (inset) for sphere and cube. We find that the vortex state undergoes several transitions for

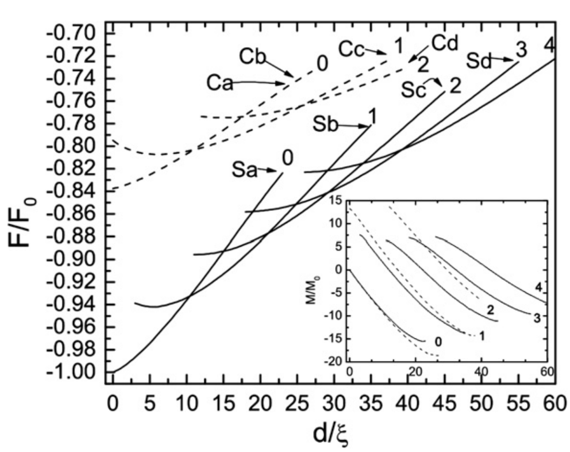


Fig. 1. Free energy vs. magnetic moment for the mesoscopic (continuous) sphere and (dashed) cube. The cube free energy is shifted upwards by an overall constant with respect to the sphere for clarity. The inset shows the induced magnetization and branches are labeled by their number of external vortex pairs. The vortex state for the selected points are shown in the next figures.

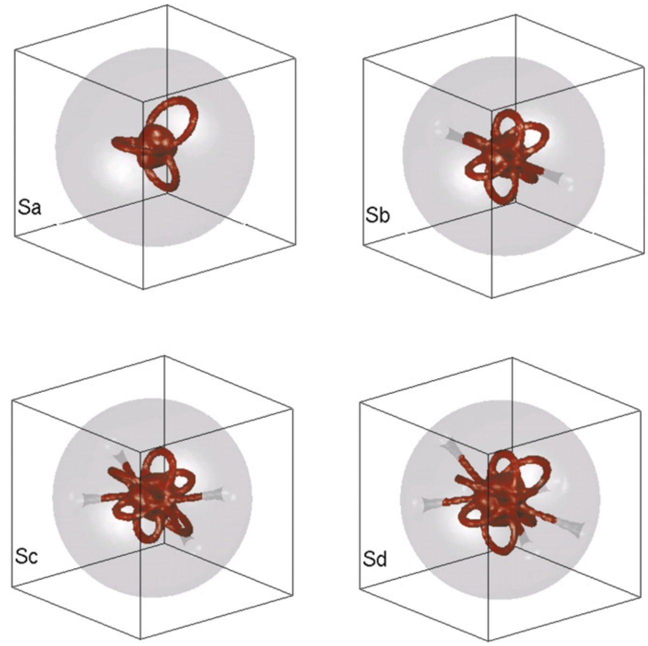


Fig. 2. Three-dimensional iso-density plots for selected sphere states.

increasing $\mu$. The diamagnetic response of the mesoscopic superconductor is hindered by $\mu$ in the total magnetization: $\mu/\mu_0 + M/M_0$. An overall normalization constant is introduced here as we assume the existence of an asymptotic Meissner phase $\mu/\mu_0 + M/M_0 = 0$ for very small $\mu$. The induced moment weakens for large $\mu$ because of the EVPs, and this renders $\mu/\mu_0 + M/M_0 > 0$. Notice that Fig. 1 shows that both magnetization and free energy split into distinct branch lines for both cube and sphere, and these branches are associated to the number of EVPs. There is strong metastability between distinct branches because EVPs are surface pinned. Fig. 1 is limited to three and four EVPs for the cube and sphere, respectively. However as $\mu$ increases along a selected branch, although the number of EVPs remains constant, the number of CVLs does not. New CVLs arise along the way and can alter the symmetry of the final state. Thus as the CVLs approach the surface, the surface symmetry can affect the state. For the cube this means that the threefold symmetry is replaced by a fourfold symmetry.

We have selected a set of vortex states to illustrate some of the features of the present system in Fig. 1, according to their $d/\xi$ values: 23(Sa), 35(Sb), 45(Sc), 55(Sd), and 60(Se) for the sphere and 23(Ca), 24(Cb), 38(Cc) and 40(Cd) for the cube. [1]Figs. 2 and 3 display density isosurfaces that show the spatial location of the vortices. Each isosurface corresponds to a single surface taken at 20% of the maximum density and is decomposed here in two parts. The external surface part stands near the finite superconductor external border and the internal surface part due to the

[1] (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of this article.)