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Critical parameters in the superconducting singular Fermi liquid model

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Abstract

Model of superconductivity in a singular Fermi liquid with divergent scattering amplitude for particles with the same momenta is studied. The model reveals some features of a pseudogap-type behaviour in the supercritical temperature range. The thermodynamics in the vicinity of the superconducting phase transition is discussed in detail. Critical parameters related to the external magnetic field and the superflow, such as the critical magnetic field and the critical current are also calculated.

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Keywords: Singular Fermi liquid; Critical parameters; Superconducting phase transition

1. Introduction

A common property of many temperature-doping phase diagrams of high T_c superconductors in the underdoped regime is the presence of the pseudogap region. In this region—especially for structures near to the optimal doping level—spectroscopic techniques reveal various anomalies [1], indicating that some features of the superconducting state persist also above the critical temperature. Therefore the normal state of optimally doped structures should be treated as a more composed form of a quantum liquid than the Fermi liquid [2–6]. Recently a wide class of non-Fermi liquids has been presented, and it has been argued that some form of a singular or marginal Fermi liquid behaviour was observed in copper-oxides in underdoped and overdoped regimes near to the structure of optimal doping [6].

In the present paper, the critical behaviour of a singular Fermi liquid (SFL) with divergent scattering amplitude for particles with the same momenta is discussed. In particular, the supercritical temperature range with a pseudogap-type behaviour is considered, as are critical parameters related to the external magnetic field and the superflow. Throughout the paper $\hbar = c = k_B = 1$.

2. Singular Fermi liquid model

The effective SFL Hamiltonian, introduced and substantiated in Refs. [7–10], includes the dispersion relation $\varepsilon_{\mathbf{k}}$ (obtained e.g. from the 2D tight-binding band model), a separable pairing potential $-\eta\zeta_{\mathbf{k}}\zeta_{\mathbf{k}'}$ and the term $R_{\mathbf{k}}n_{\mathbf{k}\uparrow}n_{\mathbf{k}\downarrow}$ responsible for scattering of particles with the same momenta $\mathbf{k}$ and opposite spins. For an arbitrary value of the scattering amplitude $R_{\mathbf{k}}$, the Hamiltonian acts in a 16-dimensional Fock space spanned upon the basis $\psi_{\sigma\sigma'}(\mathbf{k}) = \left|\begin{matrix}\mathbf{k},\sigma\\ -\mathbf{k},\sigma'\end{matrix}\right\rangle$ where σ, $\sigma' = 0$, $\uparrow$, $\downarrow$ or $\uparrow\downarrow$. However, when the scattering amplitude is divergent ($R_{\mathbf{k}} \to \infty$) the doubly occupied states $\psi_{\sigma\uparrow\downarrow}(\mathbf{k})$ and $\psi_{\uparrow\downarrow\sigma'}(\mathbf{k})$ become energetically forbidden and the Fock space effectively reduces to a 9-dimensional one with the basis $\psi_{\sigma\sigma'}(\mathbf{k})$, where σ, $\sigma' = 0$, $\uparrow$, $\downarrow$. Consequently the SFL Hamiltonian acting in the reduced space reads

$$H_{\mathrm{SFL}} = \sum_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}}(n_{\mathbf{k}\uparrow} + n_{\mathbf{k}\downarrow}) - \eta \sum_{\mathbf{k},\mathbf{k}'} \zeta_{\mathbf{k}}\zeta_{\mathbf{k}'} a^{+}_{\mathbf{k}\uparrow} a^{+}_{-\mathbf{k}\downarrow} a_{-\mathbf{k}'\downarrow} a_{\mathbf{k}'\uparrow}, \qquad (1)$$

where $\tilde{\varepsilon}_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \mu$ and μ denotes the chemical potential.

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Although it is possible to obtain all results in quite general forms [7], in order to perform some numerical calculations, one considers the case of a purely s-wave paired SFL with the dispersion relation $\varepsilon_{\mathbf{k}} = \xi = v_F(k - k_F)$, pairing coupling $\eta\zeta_{\mathbf{k}}\zeta_{\mathbf{k}'} = \lambda$, and an isotropic order parameter $\Delta_{\mathbf{k}} = \Delta(T)$. Moreover it is assumed that the density of states $N(\xi) \approx N(0)$, i.e. is constant in the nearly half-filled conduction band of the width 2ω. The results are presented for three chosen values of the band-width.

3. Thermodynamics in the critical temperature region

Thermodynamics of the system described by the Hamiltonian (1) has been studied within the Bogolubov method [11]. In particular the free energy of both phases: normal and superconducting, have been found as well as other thermodynamic characteristics such as the entropy and the specific heat [7–9]. In the present section a detailed discussion of the critical temperature region is presented for the case of the SFL with purely s-wave pairing [7].

In Fig. 1 solutions of two fundamental equations obtained from the minimalization condition for the free energy are presented, namely the gap equation and the equation for the chemical potential. Analysis of the results presented in Fig. 1 allows one to state that in the temperature range $0 \leqslant T < T_0$ the gap equation (and the equation for the chemical potential, consequently) has one solution. At temperature $T = T_0$ another solution emerges, opening the second, lower branch of the energy gap (chemical potential) as displayed in Fig. 1. Denoting by T^* the highest value of the temperature for which a non-zero solution of the gap equation (and the equation for the chemical potential) exists, one can conclude that in the temperature range $T_0 \leqslant T < T^*$ the gap (chemical potential) equation has two solutions. In order to find out for which one of these solutions the free energy of the superconducting phase is lower than the free energy of the normal phase (i.e. which solution and in what temperature range is realized in the superconducting system), the free energy values for both solutions have been calculated and compared with the free energy of the normal phase [7,8]. It has turned out that the free energy corresponding to the lower branch of the energy gap (chemical potential) is greater than the free energy of the normal phase in the whole temperature range $T_0 \leqslant T < T^*$ where the second solution exists [7,8]. For the higher branch, the free energy of the superconducting phase is less than the free energy of the normal phase for temperature $0 \leqslant T < T_c$. When $T_c < T \leqslant T^*$ the relation changes—the normal phase becomes the phase of the lower free energy—and the system is not superconducting anymore. The temperature T_c at which the values of the free energy for both phases are equal, $F_S(T_c) = F_N(T_c)$, is the temperature of the first-order phase transition between the superconducting and the normal phase [7–9]. Note that, because for $T_c < T \leqslant T^*$ the free energy corresponding to the higher branch of the energy gap (and chemical potential) is greater than the free energy of the normal phase, in this temperature region the superconducting state can be only a metastable one (cf. Fig. 2). The values of all three characteristic temperatures fulfilling the relation $T_0 < T_c < T^*$ are quoted in Table 1.

The presented results evidence that the system displays some features of the superconducting state also in the supercritical temperature range, $T_c < T \leqslant T^*$. The non-zero value of the energy gap points at a possibility of Cooper pairs formation, although in terms of the free energy the preferred phase is the normal one. The supercritical region with a non-zero energy gap (cf. Fig. 1 and Table 1) can be related to the emergence of the pseudogap. Then in the SFL system the following scenario can be realized: At the

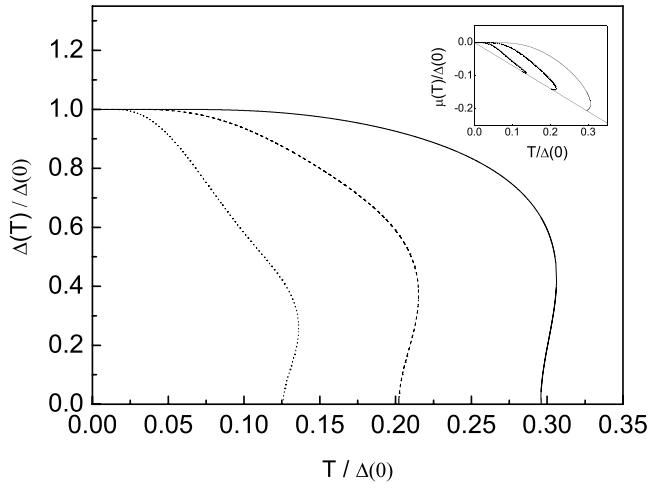

Fig. 1. Solutions of the gap equation and the equation for the chemical potential (inset) for $\omega/\Delta(0) = 2$ (solid), $2\sqrt{5}$ (dash), 10 (dot).

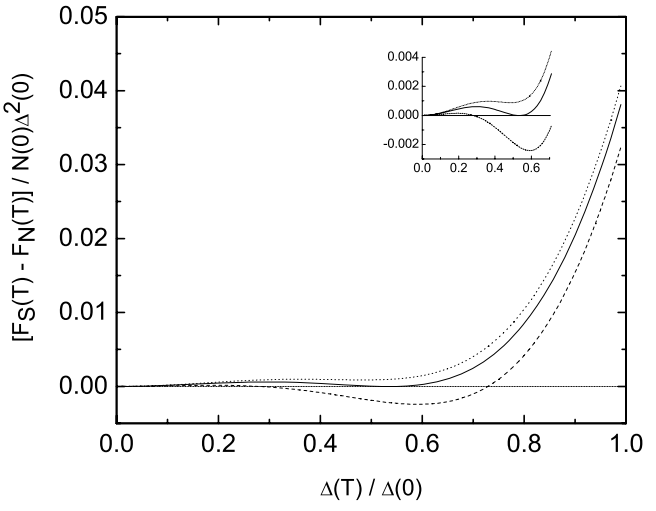

Fig. 2. Free energy difference as a function of the energy gap for $\omega/\Delta(0) = 2$. For each given value of the energy gap the chemical potential has been calculated. The calculations have been performed for three temperature values: subcritical $T_1/\Delta(0) = 0.3$ (dash), critical $T_c/\Delta(0) = 0.3038$ (solid) and supercritical $T_2/\Delta(0) = 0.3055$ (dot), fulfilling the relation $T_0 < T_1 < T_c < T_2 < T^*$. For the temperature T_2 a metastable state in the pseudogap region is possible. The horizontal line indicates the zero level.