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Vortex structure in weak to strong coupling superconductors: Crossover from BCS to BEC

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Abstract

High-T_c cuprate and MgB_2 superconductors are recognized as intermediate or strong coupling superconductors. In such a non-weak regime, the vortex core structure is an interesting issue, since the vortex dissipation is expected to be anomalously reduced due to large level distance between localized core states. In this paper, we systematically clarify the vortex core structure based on the fermion–boson model which can cover a full coupling range from weak BCS to strong BEC superconductors. The mean-field calculations on the model reveal that the distances between the low-lying core levels expand and the low-lying states finally disappear when changing the coupling from weak BCS to strong BEC regime. This result suggests that the strong coupling makes the vortex motion non-dissipative.

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1. Introduction

Since the discovery of high-T_c cuprate superconductors and metallic alloy superconductor MgB_2 [1], their vortex core structures have attracted much interest [2,3]. This is because these have been recognized as superconductors which belong to intermediate or strong coupling regime. In weak coupling BCS superconductors, in which the superconducting gap energy (Δ) is much smaller than Fermi energy (E_F), the vortex core level spectrum is known to be almost continuous, and therefore, the vortex core can be regarded as "normal core" suggested by Bardeen and Stephen [4]. On the other hand, in strong coupling regime, the level spacing between the localized core states (given as $\delta E \sim \Delta^2/E_F$ [5]) expands, and the quantum discrete features are expected to appear. Although such features are believed to be reflected in the vortex dissipation, the relationship between the coupling strength and the core states has not yet been established due to lack of systematic investigation. Therefore, to systematically explore the vortex core from weak to strong coupling regime is currently a very interesting issue. In this paper, we clarify the vortex core structure based on a model which can cover a full range from weak BCS to strong BEC superconductivity. The model is the fermion–boson model, which has been intensively investigated in atomic Fermi gases very recently. In atomic Fermi gases, the Feshbach resonance is a key phenomenon, which enables to tune their superfluidity from weak BCS-like to strong BEC-like one. Theoretically, this is given by shifting the threshold energy of the Feshbach resonance onto the chemical potential in the fermion–boson model. In this paper, we employ the fermion–

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boson model and study how the vortex core structure changes by solving BdG equation derived from the model. The results are quantitatively applicable for the atomic Fermi gases, while the systematic results give a qualitative picture on how the superconducting vortex core changes with the coupling strength.

2. The fermion–boson model and the BdG equation

The Hamiltonian of the fermion–boson model for the atomic Fermi gas is given as [6]

$$\begin{aligned} H_{\mathrm{BF}} = \int \mathrm{d}\mathbf{r}\Big[&\psi_\sigma^\dagger(\mathbf{r})\left(-\frac{1}{2m}\nabla^2-\mu\right)\psi_\sigma(\mathbf{r}) \\ &- U\psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r}) \\ &+ \varphi_{\mathrm{B}}^\dagger(\mathbf{r})\left(-\frac{1}{4m}\nabla^2+2v-2\mu\right)\varphi_{\mathrm{B}}(\mathbf{r}) \\ &+ g(\varphi_{\mathrm{B}}^\dagger(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})+\varphi_{\mathrm{B}}(\mathbf{r})\psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r}))\Big], \end{aligned} \tag{1}$$

where $\psi_\sigma(\mathbf{r})$ and $\varphi_{\mathrm{B}}(\mathbf{r})$ are the field operators of the fermion with spin $\sigma=\uparrow,\downarrow$ and the pair bosons, respectively, and U, μ, and v are the BCS attractive interaction ($U>0$), the chemical potential, and the threshold energy of the Feshbach resonance, respectively. The last term with the coupling constant g in Eq. (1) represents the process in which a boson is created from two fermion atoms and vice versa due to the Feshbach resonance [6]. In the model, when $v>E_{\mathrm{F}}$ (Fermi energy, $\sim\mu$ in BCS) the pair boson is a minority and BCS weakly coupled superfluid emerges. At $v=E_{\mathrm{F}}$, the fermions and the pair bosons coexist. When $v<E_{\mathrm{F}}$ the majority becomes pair boson, and BEC of pair bosons occurs [6]. This change from BCS to BEC is a crossover, which is realized by shifting the threshold parameter in the fermion–boson model (1).

In the mean-field approximation, the gap functions are given by the vacuum expectation values as

$$\phi^{\mathrm{B}}(\mathbf{r}) = g\langle\varphi_{\mathrm{B}}(\mathbf{r})\rangle, \tag{2}$$

$$\Delta^{\mathrm{F}}(\mathbf{r}) = U\langle\psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\rangle \tag{3}$$

and the generalized BdG equation [7,8] is derived from Eq. (1) as

$$\begin{pmatrix} H_\sigma & \Delta^{\mathrm{F}}(\mathbf{r})+\phi^{\mathrm{B}}(\mathbf{r}) \\ \Delta^{\mathrm{F}*}(\mathbf{r})+\phi^{\mathrm{B}*}(\mathbf{r}) & -H_\sigma^* \end{pmatrix}\begin{pmatrix} u_{n\sigma} \\ v_{n\sigma}\end{pmatrix} = E_n\begin{pmatrix} u_{n\sigma} \\ v_{n\sigma}\end{pmatrix}, \tag{4}$$

where $H_\sigma \equiv -\frac{1}{2m}\nabla^2-\mu$. Similarly, an equation for $\phi^{\mathrm{B}}(\mathbf{r})$ is obtained as

$$\left[-\frac{1}{4m}\nabla^2+2v-2\mu\right]\phi^{\mathrm{B}}(\mathbf{r}) = \frac{g^2}{U}\Delta^{\mathrm{F}}(\mathbf{r}). \tag{5}$$

These equations should be solved self-consistently together with the BCS gap equation

$$\Delta^{\mathrm{F}}(\mathbf{r}) = U\sum_n u_n(\mathbf{r})v_n^*(\mathbf{r}). \tag{6}$$

Here, we note that above theoretical framework neglects effects of the pairing and other fluctuations. However, the vortex solution including the quasi-particle structure around the vortex core is valid at zero temperature [7,8].

3. Numerical calculation for a single vortex

Let us study an isolated vortex of 2D s-wave case according to the calculation method given in [9]. By expressing the gap functions as $\Delta^{\mathrm{F}}(\mathbf{r})=\Delta^{\mathrm{F}}(r)\mathrm{e}^{-\mathrm{i}\theta}$ and $\phi^{\mathrm{B}}(\mathbf{r})=\phi^{\mathrm{B}}(r)\mathrm{e}^{-\mathrm{i}\theta}$ in the 2D cylindrical coordinates $\mathbf{r}=(r,\theta)$, the eigenfunctions $u_n(\mathbf{r})$ and $v_n(\mathbf{r})$, can be expanded as

$$\begin{aligned} u_{n,\eta}(\mathbf{r}) &= \sum_i c_{n,i}\phi_{i,\eta-1/2}(r)\exp[\mathrm{i}(\eta-1/2)\theta] \\ v_{n,\eta}(\mathbf{r}) &= \sum_i d_{n,i}\phi_{i,\eta+1/2}(r)\exp[\mathrm{i}(\eta+1/2)\theta], \end{aligned} \tag{7}$$

where $\phi_{i,m}(r)\equiv[\sqrt{2}/RJ_{m+1}(\alpha_{im})]J_m(\alpha_{im}r/R)$, $|\eta|=1/2,3/2,5/2,\ldots$, and i is a positive integer ($\leqslant M$), depending on the value η [9]. Thus, the generalized BdG equation can be solved as an eigenvalue problem for $2M(\eta)\times 2M(\eta)$ matrices [9]. Throughout this paper, the energy and the radial distance are normalized by unit of E_{F} and $1/k_{\mathrm{F}}$, respectively. The coupling constants are fixed to be $U=0.5$ and $g=0.6$, and R is taken as $R=40\frac{1}{k_{\mathrm{F}}}$. Here, we note that g in real atomic Fermi gases shows a large value compared to E_{F} ($g\gg E_{\mathrm{F}}$). Such a resonance ($g\ll E_{\mathrm{F}}$) is called "broad Feshbach resonance", while the present case ($g=0.6$) is assigned as "narrow Feshbach resonance". The narrow Feshbach resonance is considered to be more realistic as a model of superconductor.

4. Superconducting gap profile

Fig. 1 presents radial distributions of the superfluid gap function. In this figure, it is noted that we separate the superfluid gap function into two components, i.e., the BCS gap function Δ and the bound pair function ϕ^{B}. These three panels (a), (b), and (c) of Fig. 1 display typical radial distributions of the superfluid gap functions in three regimes, i.e., BCS ($v=3.0$), crossover ($v=1.1$), and BEC regime ($v=0.1$), respectively. When $\mu\sim1.0$, the excitation level of the bound pair approaches to zero, and the population of the bound pairs grows. Thus, the system shows the crossover from BCS to BEC at $v\sim1$. It is found from Fig. 1(a) and (b) that the core depression region of the BCS gap function shrinks when approaching from BCS to crossover regime. Such a tendency is clear when we look at the recovery point to the bulk amplitude as pointed by vertical (red) arrows in Fig. 1(a) and (b).[1] This feature is reasonable because the coupling becomes effectively strong with approaching to the crossover, and therefore, the coherence length shrinks. In fact, this tendency on the length scale

[1] For interpretation of colour in Fig. 1, the reader is referred to the web version of this article.