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Mode-locking and dynamical correlation effects in a driven Josephson junction network

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Abstract

Mode-locking phenomena in a Josephson junction network (JJN) driven by both dc and ac currents are studied using a numerical simulation. We consider a ladder type of structure of the JJN, i.e., Josephson junction ladder (JJL). Under a random magnetic field, it is found that the mode locking exists in the present system, and mode-locked steps are observed in the dc current–voltage characteristics. Outside the steps, due to the effect of the random field, plastic deformation of phase–phase coupling occurs, and then there appears plastic flow. As the bias current is increased further, transition from the plastic to elastic flow state occurs. Around the transition point, a critical behavior of the voltage–voltage correlation is observed. We discuss the peculiar nature of the mode-locking phenomena in the JJL.

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1. Introduction

Mode-locking phenomena have been observed in single Josephson junction systems driven by both a dc and an ac bias, and the physical properties have been studied thoroughly using both experimental and theoretical methods [1,2]. Shapiro steps in current–voltage curves are characteristic of the mode-locking. In the case of Josephson junction networks (JJN), which consist of arrays of superconducting sites coupled with Josephson junctions, there appear diverse mode-locking phenomena such as fractional giant Shapiro steps [3]. There remain complicated unresolved problems concerning the mode-locking of JJNs.

In this study, we consider a ladder type of JJN, which is called a Josephson junction ladder (JJL), in a random magnetic field [4,5]. We investigate mode-locking phenomena and related dynamical correlation effects in the JJN. We clarify the physical mechanism that governs the mode-locking and -unlocking dynamics of the JJN.

2. Model of a Josephson junction network

Fig. 1 shows the JJN considered here, which consists of an array of superconducting grains. We assume a ladder type of the network structure [4,5]. The ladder has two rows parallel to the x-direction, and each row has N_x superconducting sites. Each pair of the nearest-neighbor sites is connected by a Josephson junction in both the x- and y-directions.

The lattice constants are the same in both directions. Bias currents are injected (taken out) at the upper (lower) row along the y-direction. A magnetic field B is applied in the z-direction. We consider here the current-driven resistively shunted junction (RSJ) model to analyze the time evolution of the phases of superconducting wave functions. The phases on the ith site of the upper and lower rows are denoted by ϕ_i and ϕ_i', respectively.

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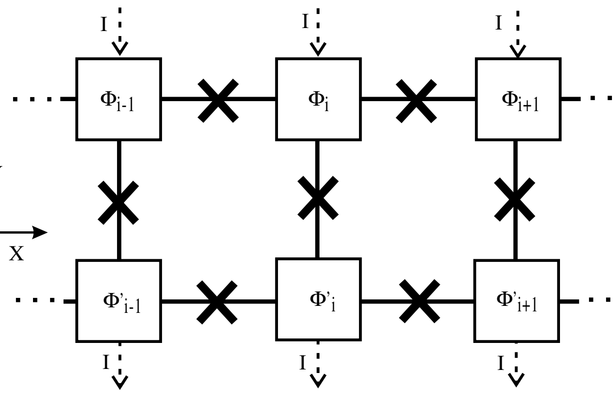

Fig. 1. Schematic sketch of a Josephson junction ladder (JJL).

In the presence system, the bias currents I has both dc and ac bias components and is given by

$$I = I_{\mathrm{dc}} + I_{\mathrm{ac}}\sin(\omega t), \tag{1}$$

where I_{dc} is the strength of the dc component, and I_{ac} and ω are the amplitude and frequency of the ac component, respectively.

In the presence of both dc and ac bias currents, the equations of motion of phases are given by

$$\frac{\hbar}{2eR}\left[3\dot{\phi}_i - \dot{\phi}_{i-1} - \dot{\phi}_{i+1} - \dot{\phi}'_i\right] = I + J_{\mathrm{c}}\Big[\sin(\phi_{i-1} - \phi_i - A_{i,i-1}) + \sin(\phi_{i+1} - \phi_i - A_{i,i+1}) + \sin(\phi'_i - \phi_i - A_i)\Big], \tag{2}$$

and

$$\frac{\hbar}{2eR}\left[3\dot{\phi}'_i - \dot{\phi}'_{i-1} - \dot{\phi}'_{i+1} - \dot{\phi}_i\right] = -I + J_{\mathrm{c}}\Big[\sin(\phi'_{i-1} - \phi'_i - A'_{i,i-1}) + \sin(\phi'_{i+1} - \phi'_i - A'_{i,i+1}) + \sin(\phi_i - \phi'_i + A_i)\Big], \tag{3}$$

where J_{c} is the critical current of junctions, and R the junction resistances. The line integrals of the vector potential $(2\pi/\Phi_0)\int_r^{r'} \mathbf{A}\cdot d\mathbf{l}$ are denoted by $A_{i,j}$ along the x-direction of the upper row, $A'_{i,j}$ along the x-direction of the lower row, and A_i along the y-direction between the upper and lower rows.

We consider a random gauge in the vector potential [4,5]. The line integrals of $\mathbf{A}$ along the y-direction, A_i, are considered as a random number distributed in $[-\pi, \pi]$, and all terms along the x-direction are set to be zero: $A_{i,j} = A'_{i,j} = 0$. Under this random gauge condition, we can consider a random magnetic field, which corresponds to a strong disorder limit of JJLs under a uniform magnetic field vertical to the xy-plane. The present model is called the random gauge (or gauge glass) RSJ model. In the following we show results for a fixed random configuration of A_i. The results are essentially independent of the random configuration under the condition of uniform distribution of A_i.

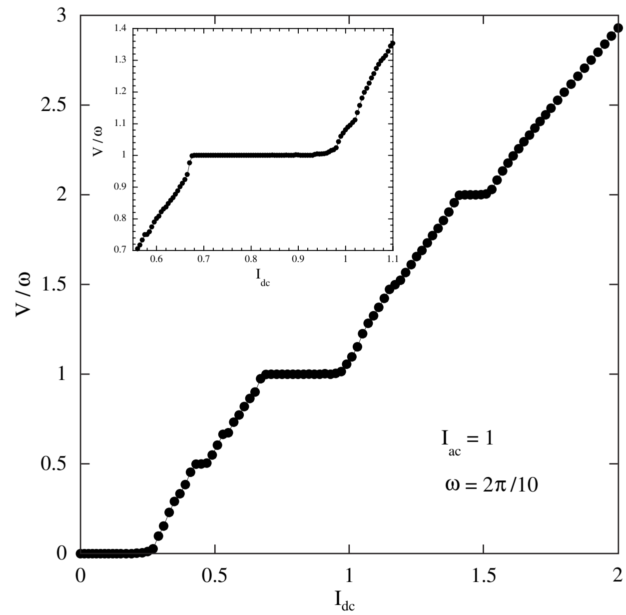

Fig. 2. DC current–voltage characteristics under ac bias currents.

We assume periodic and free boundary conditions in the x- and y-directions, respectively. We set the values of parameters as $N_x = 200$, $2e/\hbar = 1$, $R = 1$, $I_{\mathrm{ac}} = 1$ and $\omega = \frac{2\pi}{10}$. Numerical simulations of Eqs. (2) and (3) are performed using a Runge–Kutta algorism employed in [5,6].

3. Current–voltage characteristics

We first investigate the dc current–voltage characteristics. Fig. 2 shows the time-averaged voltage drops across the JJN in the y-direction along the bias current,

$$V = \frac{1}{N_x}\sum_i \bar{\dot{\theta}}_i, \tag{4}$$

where θ_i $(= \phi_i - \phi'_i)$ is the phase difference between ϕ_i and ϕ'_i at site i.

There appear steps in the I_{dc}–V curve. The voltages of main steps are given by $V_n^{\mathrm{ML}} = n\omega$, where n is an integer. This result shows the existence of the mode-locking in the present JJN subject to a random magnetic field.

The inset of Fig. 2 shows an enlarged I_{dc}–V curve around the $n = 1$ step. It is found that the I_{dc}–V curve at the edge of the step shows complicated behavior. This suggests that the unlocking from the locked step is affected by complex dynamics of the JJN. In the present JJN system, the process of unlocking has not been examined yet and remains unclear. In the next section, we clarify the unlocking dynamics.

4. Plasticity of phase–phase coupling

To clarify the mechanism from the mode-locking to unlocking, we investigate local voltage drops at each site. In Fig. 3, the time-averaged $\dot{\theta}_i$ which is normalized with