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Ponderomotive instabilities and microphonics—a tutorial ☆

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Abstract

Phase and amplitude stabilization of the fields in superconducting cavities in the presence of ponderomotive effects and microphonics was one of the major challenges that had to be surmounted in order to make superconducting rf accelerators practical. This was of particular concern in low-velocity proton and ion accelerators since the beam loading was often negligible, but was usually not relevant in electron accelerators since the beam loading was often high and the gradients low. More recent or future applications of electron linacs—for example JLab upgrade, energy recovering linacs (ERLs)—will operate at increasingly higher gradients with little beam loading, and the issues associated with microphonics and ponderomotive instabilities will again become relevant areas of research. This paper will describe the ponderomotive instabilities and the conditions under which they can occur, and review the methods by which they, and microphonics, can be overcome.

Keywords: Superconducting RF cavities; Microphonics; Ponderomotive effects; Instabilities; RF control

1. Historical background

Ponderomotive instabilities were first observed in normal conducting resonators in the 1960s in the Soviet Union [1–3]. In that work stability conditions were derived using energetic arguments, comparing the rate of transfer of energy from the electromagnetic mode to the mechanical mode and the rate of dissipation of energy of the mechanical mode. The analysis was valid when the decay time of the electromagnetic mode was much less that the period of the mechanical mode ($\tau\Omega_\mu \ll 1$), or when the rate of transfer of energy was very high.

In the late 60s early 70s, as part of the R&D activities at Karlsruhe toward the development of a superconducting proton accelerator, Schulze [4,5] extended the analysis of ponderomotive instabilities in generator-driven systems, with and without phase and amplitude feedback, to arbitrary $\tau\Omega_\mu$, which would be appropriate for superconducting structures. His analysis was based on control system methods (Laplace transforms, transfer functions, etc.). That work made first mention and demonstrated the effectiveness of using ponderomotive effects to damp mechanical modes.

In the mid-70s, as part of the R&D activities at Caltech toward the development of a heavy-ion superconducting accelerator, Delayen [6,7] analyzed the behavior of resonators operated in self-excited loops, with and without phase and amplitude feedback, in the presence of ponderomotive effects and microphonics. That analysis was also based on control systems methods and made use of stochastic analysis to quantify the performance of the feedback systems. That work also introduced the I/Q control method as well as microprocessor-based control systems for superconducting cavities.

2. The adiabatic theorem and superconducting cavities

An important theorem of classical mechanics states that for periodic system whose properties change slowly with time (as defined by a slowness parameter ε) the action $J = \oint p\,dq$ changes more slowly than a power of ε. When applied to harmonics oscillators—where the action is $\frac{U}{\omega}$,

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the ratio of energy and frequency—then $\frac{U}{\omega}$ changes more slowly than any power of $\varepsilon = \frac{1}{\omega^2}\frac{d\omega}{dt}$ if the frequency changes smoothly, i.e. it is an adiabatic invariant to all orders [8]. The dimensionless parameter ε is the relative change in frequency during one radian. Since in the case of superconducting cavities it would be difficult to change the frequency significantly during one radian, the action $\frac{U}{\omega}$ can be assumed to be constant and, in particular, any relative change in frequency is equal to any relative change in energy content: $\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U}$.

In the quantum picture, this would mean that the system stays in the same eigenstate and that the number of photons remains constant ($U = N\hbar\omega$).

The energy content in a resonator is given by

$$U = \int_V \left[\frac{\mu_0}{4}H^2(\vec{r}) + \frac{\varepsilon_0}{4}E^2(\vec{r})\right] dv \tag{1}$$

and the change in energy content is equal to the work done by the radiation pressure:

$$\Delta U = -\int_S dS\,\vec{n}(\vec{r})\cdot\vec{\xi}(\vec{r})\left[\frac{\mu_0}{4}H^2(\vec{r}) - \frac{\varepsilon_0}{4}E^2(\vec{r})\right] \tag{2}$$

where $\vec{n}(\vec{r})$ and $\vec{\xi}(\vec{r})$ are the normal vector and the displacement vector, respectively, at location $\vec{r}$.

The relative change in frequency is then given by

$$\frac{\Delta\omega}{\omega} = -\frac{\int_S dS\,\vec{n}(\vec{r})\cdot\vec{\xi}(\vec{r})\left[\frac{\mu_0}{4}H^2(\vec{r}) - \frac{\varepsilon_0}{4}E^2(\vec{r})\right]}{\int_V\left[\frac{\mu_0}{4}H^2(\vec{r}) + \frac{\varepsilon_0}{4}E^2(\vec{r})\right]} \tag{3}$$

which, in microwave engineering, is known as Slater's formula [9].

3. Ponderomotive effects

Any cavity will have an infinite number of mechanical eigenmodes of vibration represented by a complete infinite set of orthonormal displacement functions $\phi_\mu(\vec{r})$. The actual displacements of the cavity wall, $\xi(\vec{r})$ and the forces on the wall, $F(\vec{r})$ can be expanded into the functions $\phi_\mu(\vec{r})$:

$$\xi(\vec{r}) = \sum_\mu q_\mu\phi_\mu(\vec{r}), \quad q_\mu = \int_S \xi(\vec{r})\phi_\mu(\vec{r})\,dS$$
$$F(\vec{r}) = \sum_\mu F_\mu\phi_\mu(\vec{r}), \quad F_\mu = \int_S F(\vec{r})\phi_\mu(\vec{r})\,dS \tag{4}$$

where q_μ is the amplitude of mechanical mode μ whose equation of motion is

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{q}_\mu} - \frac{\partial L}{\partial q_\mu} + \frac{\partial\Phi}{\partial\dot{q}_\mu} = F_\mu \tag{5}$$

with $L = T - U$, where U, T, and Φ are the kinetic energy, the potential energy, and the power dissipation, respectively.

$$U = \frac{1}{2}\sum_\mu c_\mu q_\mu^2, \quad T = \frac{1}{2}\sum_\mu c_\mu\frac{\dot{q}_\mu^2}{\Omega_\mu^2}$$
$$\Phi = \sum_\mu \frac{c_\mu}{\tau_\mu}\frac{\dot{q}_\mu^2}{\Omega_\mu^2} \tag{6}$$

where c_μ is the elastic constant, Ω_μ is the frequency, and τ_μ is the decay time of mechanical mode μ. Eq. (5) then becomes

$$\ddot{q}_\mu + \frac{2}{\tau_\mu}\dot{q}_\mu + \Omega_\mu^2 q_\mu = \frac{\Omega_\mu^2}{c_\mu}F_\mu \tag{7}$$

Since the frequency shift $\Delta\omega_\mu$ caused by mechanical mode μ is directly proportional to q_μ, and the force F_μ due to the radiation pressure is proportional to the square of the field amplitude V, the equation for $\Delta\omega_\mu$ is

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu^2\omega_\mu = -k_\mu\Omega_\mu^2 V^2 + n(t) \tag{8}$$

The constant k_μ (the Lorentz coefficient for that mode) represents the coupling between the rf field and mechanical mode μ, and $n(t)$ is an additional driving term representing external vibrations or microphonics. The total frequency shift is

$$\Delta\omega(t) = \sum_\mu \Delta\omega_\mu(t)$$

and in steady-state

$$\Delta\omega_0 = \sum_\mu \Delta\omega_{\mu 0} = -V^2\sum_\mu k_\mu, \text{ and } k = \sum_\mu k_\mu$$

is the static Lorentz coefficient of the cavity.

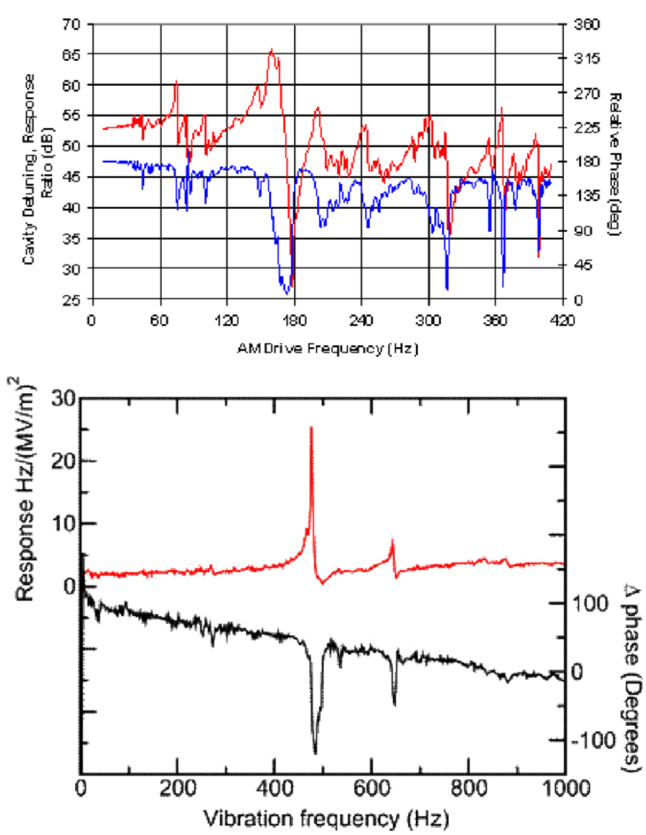

Fig. 1. Lorentz transfer function of a $\beta = 0.61$, 805 MHz 6-cell elliptical cavity (top) [10], and of a double-spoke 352 MHz, $\beta = 0.4$ cavity (bottom) [11].