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Investigation of fundamental and higher harmonics of ac susceptibility under general field direction: Effects of field and sample geometry

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Abstract

Behaviour of fundamental and higher harmonics of ac susceptibility is studied for cylindrical geometry subjected to an applied ac field $B = B_{ac}\cos(\omega t)$ along any arbitrary direction with respect to the cylinder axis. Variations of the parallel and two perpendicular in-phase (χ') and out-of-phase (χ'') ac susceptibility components are investigated in details with respect to the change in the applied field geometry (field orientations and amplitudes). The effects of geometrical anisotropy of the sample on the ac susceptibility measurements are also discussed. Some of our selected results compares well with the experimental studies.

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1. Introduction

To investigate the non-linear dynamic response of hard type-II materials, measurement of the fundamental and higher harmonics of ac susceptibility is a powerful technique [1–3]. Such measurements offer many possibilities to analyse the macroscopic properties of an wide varieties of vortex phases in type-II materials such as: the transition temperature of each structural phase in a multiphase material, investigation of irreversibility line, to measure the critical current density and pinning potential, to separate the inter-grain and intra-grain contribution in a polycrystalline sample, determination of the critical magnetic fields (H_{c1} and H_{c2}) and so on.

In the mixed state of type-II superconductors, ac losses will be present due to vortex motion. The effects of this on the magnetization properties may be either reversible or irreversible. In the reversible region of the vortex phase diagram, the ac response is linear and for the irreversible region, non-linear effects and higher harmonics becomes apparant. The irreversibility in the magnetization behaviour can be accounted in terms of bulk pinning and surface or geometrical barriers. Calculation of ac susceptibility by assuming merely bulk pinning is widely studied in the framework of critical state model (CSM) [4,5]. There are also works available in the literature dealing with geometrical surface barrier-induced irreversibility for a pin-free mixed state (neglecting bulk pinning) of thin film superconductors [6,7], where the effect of surface barriers on the higher harmonics of ac susceptibility measurements are studied. On the other hand, a large number of studies deal with the linear and non-linear response of the superconducting system, in different regions of a *E–J* characteristics [8–12]. By incorporating fluxon dynamics and solving the non-linear diffusion equation for the local flux density, the pinning potential,

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thermal activation energy and critical current values can be extracted from the ac susceptibility higher harmonics data [13,14,8].

In our study, we deal with an infinite cylinder of any arbitrary cross-section subjected to any general directional ac field $B_a = B_{ac}\cos\omega t$. The fundamental and higher harmonic response of the parallel and two perpendicular components of ac susceptibility are investigated with the variation of ac field amplitude, polar or azimuthal orientation of the applied field, i.e., with the variation of applied field geometry. The effect of sample geometry (geometrical anisotropy) on the fundamental and higher harmonic ac response of the sample is also studied by varying the a/b ratio of a rectangular cylinder for various values of the ac field amplitude. In all these cases, bulk pinning is assumed to be the basic mechanism to induce irreversibility (neglecting surface barrier) and hence our formulation is within the CSM description. Only the effects of sample and applied field geometry on the measurement of fundamental and higher harmonics of ac susceptibility can be best investigated by assuming the critical current density (J_c) to be isotropic and field independent. Henceforth, we consider the type-II material to be isotropic and the critical current J_c to be independent of the local field (Bean model) to avoid any combined effect of intrinsic and geometric anisotropy on the magnetization or ac susceptibility of the sample. Here, we utilize our earlier result of studying the magnetization property of a cylindrical sample subjected to an applied field in the general direction [15]. It may also be mentioned in passing that such a geometry pertains to a non-zero demagnetization factor. In this work, we compared some of our selected results with the experimental study of Zhang and Ong [18]. They have measured higher harmonic ac responses of a silver-sheathed B*i*-2223 tapes, as a function of the orientation of the magnetic field and explained their experimental data in terms of a scaling relation proposed by Hao and Clem [16], such that the harmonic responses are scaled as $\chi_n(H,\theta) = \chi_n(H_{red})$. The reduced field is given by $H_{red} = H(\cos^2\theta/\gamma^2 + \sin^2\theta)^{1/2}$, where γ is the anisotropic parameter. Similar scaling rule was also obtained by Blatter, Geshkenbein and Larkin [17] by mapping the results from isotropic to directly an anisotropic superconductor.

In this study, we have performed a detailed analytical calculation of harmonic response of a hard type-II superconducting cylinder subjected to an arbitrary orientation of the applied field. The behaviour obtained for higher harmonic generation have close resemblance to the experimental results shown in Refs. [18,12]. This paper is organized as follows. In the next section, the analytical formulation of the problem is discussed. For illustration, a rectangular cylinder is chosen and the effects of field and sample geometry on the fundamental and odd higher harmonics of ac susceptibility are investigated in the successive sections. The general behaviour of non-linear dynamic responses of a cylindrical hard type-II material are discussed in the concluding section.

2. Analytical formulation

In this study we consider an infinite cylinder of any arbitrary cross-section subjected to an ac field $B_a = B_{ac}\cos\omega t$ along any general direction having a polar orientation β and an azimuthal orientation α. The field orientation with respect to the sample is as shown in Fig. 1. The magnetic susceptibility tensor under such a general applied field direction is defined by the equation:

$$\begin{pmatrix} \mu_0 m_x \\ \mu_0 m_y \\ \mu_0 m_z \end{pmatrix} = \begin{pmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \tag{1}$$

To have a close comparison with the experiments, we can rewrite the above equation in terms of the parallel and the two perpendicular components of magnetization as:

$$\begin{pmatrix} \mu_0 m_\parallel \\ \mu_0 m_\perp^1 \\ \mu_0 m_\perp^2 \end{pmatrix} = \begin{pmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{pmatrix} \begin{pmatrix} B_\parallel \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

The orientations of the three components of magnetization are depicted in Fig. 1. It is obvious from the above equation that knowledge about the parallel and two perpendicular components of magnetization for a particular orientation of the applied magnetic field leads to the evaluation of the first column of the susceptibility matrix. In a similar way, applying the field along three orthogonal directions, one can evaluate all the elements of the susceptibility matrix. Under the application of an ac field the elements of the tensor are complex. Hence one can write,

$$\chi_{11} = \chi_\parallel = \chi'_\parallel + \imath\chi''_\parallel, \quad \chi_{21} = \chi_\perp^1 = \chi_\perp^{1'} + \imath\chi_\perp^{1''},$$
$$\chi_{31} = \chi_\perp^2 = \chi_\perp^{2'} + \imath\chi_\perp^{2''} \tag{3}$$

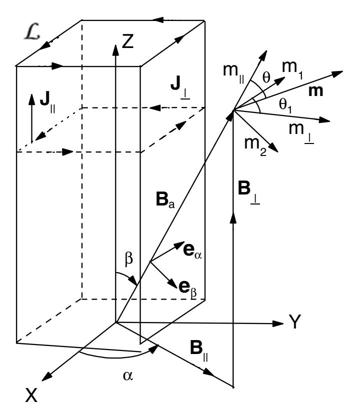

Fig. 1. The choice of the coordinate axes and the sample geometry is shown. The sample is assumed to have a rectangular cross section $\mathscr{L}$. The polar coordinates (β,α) are used to specify orientation of the applied field B_a. e_α and e_β are the unit vectors corresponding to α and β. $J_\parallel$ and $J_\perp$ are components of J parallel and perpendicular to the axis of the cyllinder. $B_\parallel$ and $B_\perp$ are components of B_a parallel to $J_\perp$ and $J_\parallel$. $m_\parallel$ and $m_\perp$ are components of magnetization along and perpendicular to the applied field. m_1 and m_2 are the two perpendicular components of magnetization along e_α and e_β.