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Switching phenomena in an annular Josephson junction

D. Gulevich *, F. Kusmartsev

Department of Physics, Loughborough University, Loughborough, LE11 3TU, UK

Abstract

We analyze the switching to resistive state of an annular Josephson junction due to thermal activation. Mostly, we concentrate on moderate length junctions with a circumference about $10\lambda_J$. Using a simple approximation the switching current distribution is analyzed and its standard deviation is analytically calculated. We show that the switching of the annular Josephson junction observed in recent experiments can be interpreted as a nearly homogeneous phase switching in the form of a classical string escape. Thus, in the regime close to the critical bias current, the behavior of the AJJ is similar to a small Josephson junction. Also, the deviation of the standard deviation from a $T^{2/3}$ power low and possible observation of the vortex–antivortex pair annihilation are discussed.

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1. Introduction

Thermal escape in Josephson devices associated with the transition from superconducting to resistive state has been studied for a very long time. Originally, it was Kramers who first estimated the probability of thermal escape from a metastable state [1]. Small Josephson junctions (JJ) are among the most studied of all superconducting devices. There two types of escape, quantum and classical, have been observed. The classical or thermal escape usually arises at high temperatures while the quantum escape manifests itself at low temperatures.

The thermal or quantum escape in small Josephson junctions is equivalent to escape of a quantum particle from a metastable state. In this paper we investigate the thermal escape of an object which has an extended dimension, i.e. very different from such point like objects like particles. Such an extended object—let us call it a string—may arise in a long annular Josephson junction (AJJ). In contrast to a small Josephson junction the phase difference $\phi(x)$ in AJJ depends on the position on the ring x. The phase difference satisfies periodic boundary conditions $\phi(0) = \phi(L)$ where L is the circumference of the AJJ. Thus, the dependence $\phi(x)$ is associated with a string of length L. The string is a one-dimensional object in contrast to a small JJ that is zero-dimensional. Similar behavior appears for dislocations in solids. The issue of the classical and quantum escape of an extended object such as a string or dislocation from a metastable state has been the subject of long-term interest [2,3], which is still under intensive discussion [4,5]. In these papers it was suggested that if the extended object is very long the escape process begins with the escape of some part of the object, namely with the nucleation of a kink–antikink pair (or a fluxon–antifluxon pair in the case of the Josephson junction). Eventually, this nucleation process gives rise to the escape of the whole object such as a long string. However, if the size of a kink–antikink pair becomes comparable with the length of the string, the escape may arise via a different mechanism without nucleation of the pair.

We study a current-biased annular Josephson junction subjected to in-plane magnetic field. In recent years this system has attracted much of attention. Existing annular

* Corresponding author. Fax: +44 1509 22 3986.
E-mail address: d.r.gulevich@lboro.ac.uk (D. Gulevich).

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junctions are favorite extended objects to study dynamics of Josephson vortices subjected to an induced periodic potential. There quantum switching of AJJ [8] has been experimentally observed and quantum dynamics of a single fluxon has been investigated in detail [7]. This prompts a possible application of AJJ for quantum computer implementation.

The use of the collective coordinate method in terms of vortex/antivortex positions is an effective tool to describe very long JJ. However, for moderate length junctions the method gives some discrepancy with experimental data in the magnetic field dependence of the switching current and the standard deviation [8]. The experiments show increase and then saturation with magnetic field while the existing theory fails to predict the correct saturation. The reason is that effective size of a vortex–antivortex pair depends crucially on the applied bias and may even exceed the length of the junction. Thus, the description in terms of vortex–antivortex positions breaks down. In the present work we consider the situation when the escape of a string from a metastable state occurs in the form of a transition of the whole object homogeneously all along the junction. In this view, the vortex–antivortex pair may develop on the later stage of the phase evolution—*after* the escape occurred. Such behavior is expected for moderate length annular junctions at low temperatures.

In our calculations we start from basic statistical field theory principles. We consider a string describing a superconducting phase difference across the junction. In classical statistical mechanics a thermal activation of a string is described as a most optimal or a most probable fluctuation. In the framework of our approach, the saturation behavior found in experiments is naturally explained. Also, the critical current dependence on magnetic field is calculated. It perfectly fits the experimental data.

2. Formulation

Our main purpose is to investigate the switching of an annular Josephson junction from superconducting to resistive state. It occurs when a parallel magnetic field and a bias voltage are applied and reach some critical value. We introduce a field $\varphi(x)$ to denote the superconducting phase difference across the junction. In this view, the analogy with classical mechanics becomes clear. The field $\varphi(x)$ describes a string in the perturbed sine-Gordon potential. We analyze thermal fluctuation of this string and find the most optimal fluctuation associated with thermal activated switching of the junction from superconducting to resistive state. We study the classical thermal activation and therefore we have to consider only stationary configurations of the phase difference $\varphi(x)$ across the junction. They are the only important ones since the other type of fluctuations give a negligible contribution in this limit. The energy of AJJ under parallel external magnetic field h and bias current γ applied may be described with the use of the following expression (we use normalized units):

$$V[\varphi] = \int_{-L/2}^{L/2} \mathrm{d}x \left[\frac{\varphi_x^2}{2} + (1-\cos\varphi) + \gamma\varphi + h\varphi\cos kx\right] \tag{1}$$

where $k = 2\pi/L$ and L is the circumference of the annular junction. The classical configurations associated with the extremal points of the energy functional, Eq. (1) are given by the perturbed sine-Gordon equation (e.g. [10,11]):

$$\varphi_{xx} - \sin\varphi - \gamma - h\cos kx = 0 \tag{2}$$

with periodic boundary conditions

$$\begin{cases} \varphi(-L/2) = \varphi(L/2) \\ \varphi_x(-L/2) = \varphi_x(L/2) \end{cases} \tag{3}$$

With such boundary conditions the total number of Josephson vortices trapped in the junction is constrained to zero. Close to the critical field configuration we make the following decomposition $\varphi_c(x) \equiv -\pi/2$ at $h = 0$ and introduce a new field variable $\xi(x) = \varphi(x) + \pi/2$. We expect that this decomposition works well in the range $|\xi(x)| \lesssim \pi/2$, since in this region the value $\sin\xi \simeq \xi - \xi^3/6$ differs by less than 10% there [6]. For moderate length junctions $L \simeq 10\lambda_J$ this requirement is satisfied when $h \lesssim 1$ as can be checked at the end of the calculations. Also, we introduce a new bias parameter $\delta = 1 - \gamma < 1$. With the use of this new variable the energy functional takes the form, which is equivalent to the previous one, up to an arbitrary constant

$$V[\xi] = \int_{-L/2}^{L/2} \mathrm{d}x \left[\frac{\xi_x^2}{2} + \frac{\xi^3}{6} - \delta\xi + h\xi\cos kx\right] \tag{4}$$

while the equation of motion is

$$\xi_{xx} - \frac{\xi^2}{2} + \delta - h\cos kx = 0 \tag{5}$$

with periodic boundary conditions implied. We use the variational method where our variational function or an approximate solution [11] is taken in the form $\xi(x) = \sqrt{2}A\cos kx + B$. Substituting this function into (4) we get an expression for the energy as a function of two variables A and B:

$$V[\sqrt{2}A\cos kx + B] = \frac{L}{2}\left(A^2(B+k^2) + \frac{1}{3}B^3 - 2B\delta + \sqrt{2}Ah\right)$$

The extremal points of this function are determined by

$$\begin{cases} 2A(B+k^2) + \sqrt{2}h = 0 \\ A^2 + B^2 - 2\delta = 0 \end{cases} \tag{6}$$

Using the first equation, the energy of a classical string configurations can be reduced to

$$V_\delta(B) = \frac{L}{2}\left(-\frac{h^2}{2(B+k^2)} + \frac{1}{3}B^3 - 2B\delta\right) \tag{7}$$

with a single constraint

$$V'_\delta(B) = \frac{L}{2}\left(\frac{h^2}{2(B+k^2)^2} + B^2 - 2\delta\right) = 0 \tag{8}$$