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Vortices in thin flat superconductors with holes and slits

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Abstract

It is shown how to compute from Maxwell–London theory the static and dynamic electromagnetic properties of thin flat superconducting films of any shape, also for SQUID washers, containing vortices or no vortices.

Keywords: Vortex lattice; Type-II superconductors; Thin films; SQUID

Superconducting quantum interference devices (SQUIDs) typically have the shape of a thin-film disk or a rectangular film, with a radial slit leading to a central hole. This slit is bridged at some point by one or two "weak links" acting as Josephson junctions that can carry a limited supercurrent. A perpendicular applied magnetic field $H_a(\mathbf{r})$ generates complicated screening currents that will pass or not pass the weak link and thereby modify the voltage response to a small test current [1]. The penetration and motion of two dimensional (2D) Pearl vortices [2] cause noise in the SQUID. For SQUIDs without and with vortices, the 2D sheet current $\mathbf{J}(x,y) = (J_x, J_y)$ and the related complicated magnetic field $H_z(x,y)$ in the film plane can be calculated for any value of the 2D magnetic penetration depth $\Lambda = \lambda^2/d$ (λ = London depth, d = film thickness, $d < \lambda$) as follows [3]. See also the different methods [4,5] and similar work on slitted rings [6] and double-strip SQUIDs [7].

Since $\nabla \cdot \mathbf{J} = 0$ in the film except at small contacts, one can express $\mathbf{J}$ in terms of a scalar potential or stream function $g(x,y)$ as $\mathbf{J} = -\hat{\mathbf{z}} \times \nabla g = \nabla \times (\hat{\mathbf{z}} g) = (\partial g/\partial y, -\partial g/\partial x)$. This function $g(x,y)$ has interesting properties:

(a) $g(x,y)$ is the local magnetization or density of tiny current loops;
(b) the contour lines of $g(x,y)$ are the current stream lines;
(c) on the boundary of the film, one may put $g(x,y) = 0$ since the boundary coincides with a stream line;
(d) the integral of $g(x,y)$ over the film area is the magnetic moment of the film if $g = 0$ on its edge;
(e) the difference $g(\mathbf{r}_1) - g(\mathbf{r}_2)$ is the current that crosses any line connecting points $\mathbf{r}_1$ and $\mathbf{r}_2$;
(f) if the film contains an isolated hole or slot such that magnetic flux can be trapped in it or a current I can circulate around it, then in this hole one has $g(x,y) = \text{const} = I$ if $g(x,y) = 0$ is chosen outside the film;
(g) in a multiply connected film with n holes, n independent constants $g_1 \ldots g_n$ can be chosen for the values of $g(x,y)$ in each of these holes. The current flowing between hole 1 and hole 2 is then $g_1 - g_2$;
(h) a vortex with flux Φ_0 in the film moves in the potential $V = -\Phi_0 g(x,y)$, since the Lorentz force on a vortex is $-\mathbf{J} \times \hat{\mathbf{z}}\Phi_0 = -\Phi_0 \hat{\mathbf{z}} \times (\hat{\mathbf{z}} \times \nabla g) = \Phi_0 \nabla g(x,y) = -\nabla V$;
(i) a vortex moving from the edge of the film into a hole connected to the outside by a slit, at each position (x,y) couples a fluxoid $\Phi_0 g(x,y)/I$ into this hole, where $g(x,y)$ is the solution that has $g(x,y) = I$ in this hole (with closed slit) and $g = 0$ outside the film.

To compute $g(x,y)$ I introduce a 2D grid that spans the film area with (preferably non-equidistant) points $\mathbf{r}_i = (x_i, y_i)$ and weights w_i such that any integral is approximated by a sum: $\int d^2 r f(\mathbf{r}) \approx \sum_{i=1}^{N} w_i f(\mathbf{r}_i)$. From Ampère's law for the current density $\mathbf{j} = \mathbf{J}/d = \nabla \times \mathbf{H}$ and the

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London equation $\lambda^2\nabla\times\mathbf{j}+\mathbf{H}=0$ one obtains by integrating over the film thickness

$$H_z(x,y) = -\Lambda[\nabla\times\mathbf{J}(x,y)]\hat{\mathbf{z}} = \Lambda\nabla^2 g(x,y), \quad (1)$$

$$H_z(\mathbf{r}_i) = H_a(\mathbf{r}_i) + \sum_j Q_{ij} w_j g(\mathbf{r}_j), \quad (2)$$

$$H_a(\mathbf{r}_i) = -\sum_j \left(Q_{ij}w_j - \Lambda\nabla^2_{ij}\right) g(\mathbf{r}_j), \quad (3)$$

$$g(\mathbf{r}_i) = -\sum_j K^\Lambda_{ij} H_a(\mathbf{r}_j) \quad (4)$$

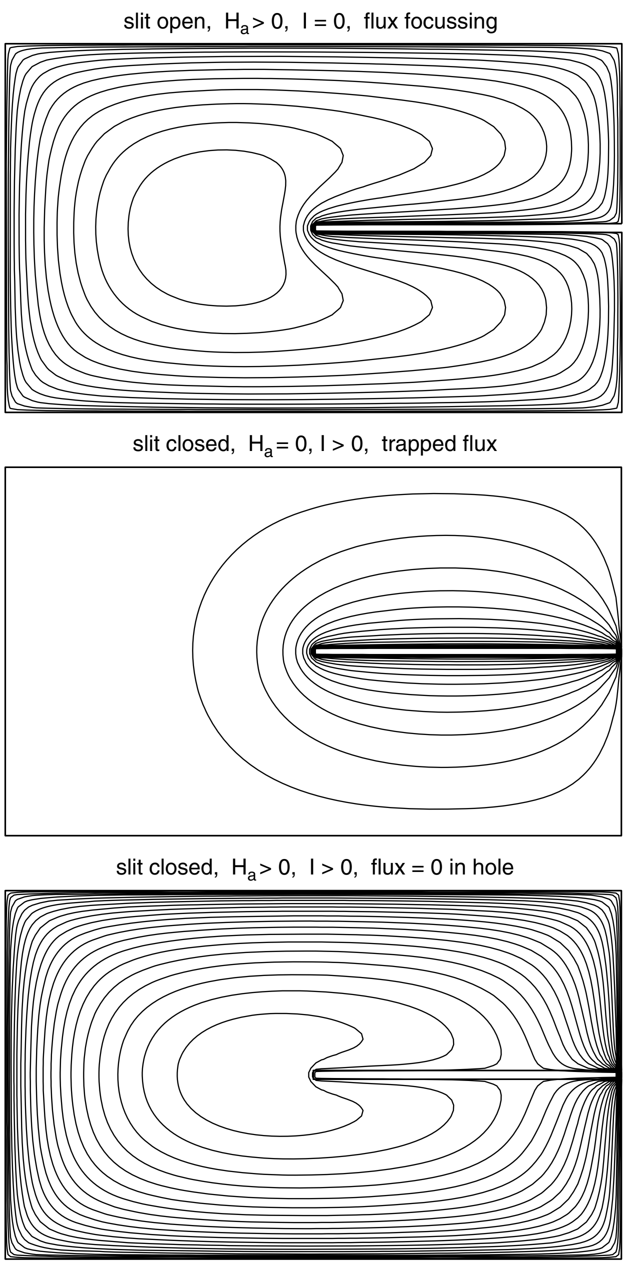

Fig. 1. Current stream lines in the ideal Meissner state $\Lambda = 0$ for a rectangular thin film ($b/a = 0.6$) with a slit. Top: Slit open, applied field $H_a > 0$, magnetic flux enters the slit and is focussed into it such that $H_z(x,y) \gg H_a$. Middle: Slit bridged at the edge, $H_a = 0$, circulating current $I > 0$ flows due to flux trapped in the slit. Bottom: Closed slit, applied field $H_a > 0$, a current $I = 0.82aH_a$ flows such that the flux in the slit is exactly zero (superposition of the two upper states).

with the inverse matrix $K^\Lambda_{ij} = (Q_{ij}w_j - \Lambda\nabla^2_{ij})^{-1}$ and the Ampère's law kernel

$$Q_{ij} = (\delta_{ij}-1)q_{ij} + \delta_{ij}\left(\sum_{l\neq i} q_{il}w_l + C_i\right)\bigg/ w_j, \quad (5)$$

where $q_{ij} = 1/(4\pi|\mathbf{r}_i-\mathbf{r}_j|^3)$, ∇^2_{ij} is the Laplacian such that $\sum_j \nabla^2_{ij} f(\mathbf{r}_j) \approx \nabla^2 f(\mathbf{r})$ at $\mathbf{r}=\mathbf{r}_i$, and

$$C_i = \frac{1}{4\pi}\sum_{p,q}\left[(a-px_i)^{-2} + (b-qy_i)^{-2}\right]^{1/2}. \quad (6)$$

Here $p,q = \pm 1$ (yielding four terms), and the grid fills the rectangle $|x| \leqslant a$, $|y| \leqslant b$ that should contain the film, i.e., the film may be this rectangle or smaller, containing holes, slits, or rounded corners.

Computation of the dynamics $\mathbf{J}(x,y,t)$, $H_z(x,y,t)$ (t = time) for any history $H_a(t)$ and given flux-motion resistivity $\rho(J)$ is described in [3,8].

The current stream lines and the stream function $g(x,y)$ in the Meissner state for various SQUID geometries are shown in Figs. 1, 2 and 4, while Figs. 3 and 5 show magnetic field profiles along the x-axis, the symmetry axis of slit and hole. Figs. 1–3 apply to a thin-film rectangle $2a\times 2b$, $b/a = 0.6$, and Figs. 4 and 5 to a square film $a = b$, slit width $\Delta \approx 0.02a$. The stream lines of the sheet current are depicted for $\Lambda = 0$ and three cases: flux focussing ($H_a = 1$, $I = 0$), flux trapping ($H_a = 0$, $I = 1$), and zero flux in hole and slit ($H_a = 1$, $I > 0$). The magnetic field profiles are shown for various Λ. For $\Lambda = 0$ (ideal screening), the enhancement of the magnetic field $H_z(x,0)$ in the slit may be understood from the estimate [9] for a long strip of width $2b$ with central slit of width $\Delta \ll b \ll a$: $H_z(x,0)/H_a \approx (2b/\Delta)/\ln(8b/\Delta) \gg 1$. For a detailed theory of two long in-plane parallel strips with arbitrary Λ in perpendicular field H_a and with applied current ("linear SQUID") see Ref. [7]. The field lines of the rectangle with slit in Fig. 1 (middle) agree with the field lines computed in Ref. [10] using the method of Ref. [5].

The above cases with closed slit consider multiply connected films. These in general have one or more holes

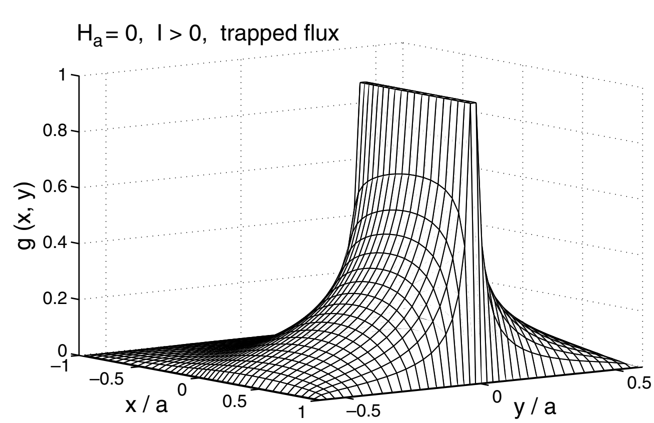

Fig. 2. The stream function $g(x,y)$ for the trapped-flux case of Fig. 1, middle, as 3D plot.