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Quasi-particle spectrum of the nano-scaled anisotropic superconducting plate

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Abstract

Using the Bogoliubov–de Gennes equation for a tight-binding model of electrons, we studied the superconducting state of nano-structured anisotropic superconductors. Assuming the attractive interaction between nearest-neighbor sites, we obtained mainly $d(\cos k_x - \cos k_y)$-wave pairing state. But the superconducting states are inhomogeneous because of the surface. Especially, square plates with $\pi/4$ rotation show suppression of the $d(\cos k_x - \cos k_y)$-wave order parameter and there appear the quasi-particle states along the surface in the superconducting energy gap.

Keywords: d-wave superconductors; Nano-scaled superconductors; Bogoliubov–de Gennes equation

1. Introduction

Recently, much attention has been focused on nano-scaled or mesoscopic superconductors. Early studies on mesoscopic superconductors were done on the confinement of the electrons and its effect on the superconductivity [1,2]. But recent studies focused on the confinement of the vortices in the superconductors with the size comparable to the coherence length or the penetration depth [3,4]. In these superconductors, the appearance of the giant vortex and anti-vortices were studied theoretically and experimentally [5,6].

Also for the superconducting network system, it is pointed theoretically that the finiteness of the system also affect on the vortex structures [7]. In the finite superconducting networks, giant vortices and anti-vortices also appear.

These previous studies were mainly focused on the conventional s-wave superconductors. For anisotropic superconductors, e.g. high-T_c superconductors, it was pointed out that effects of the boundaries are large. Tanaka and Kashiwaya showed the zero energy peak of the density of states appears at the boundary between high-T_c superconductors and conventional superconductors [8]. Matsumoto and Shiba showed that in the d-wave superconductors, different components of superconducting order parameter appear at the surface [9–11]. Therefore, it is expected that the finiteness of the anisotropic superconductor much affect its superconducting state.

Previously we studied quasi-particle spectrum of the conventional superconducting square plates under an external magnetic field, solving the Bogoliubov–de Gennes equation by the finite element methods [12,13]. In this study, we consider the model of $d(\cos k_x - \cos k_y)$-wave superconductors. Solving the Bogoliubov–de Gennes

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equation for this model, we investigate the shape dependence of their superconducting states and the quasi-particle structures.

2. Model

We start from the tight-binding Hamiltonian for square lattice

$$H = -t \sum_{\langle ij\rangle\sigma}^{\text{n.n.}} \left(C_{i\sigma}^{\dagger} C_{j\sigma} + \text{h.c.}\right) - g \sum_{\langle ij\rangle}^{\text{n.n.}} C_{i\sigma}^{\dagger} C_{j\sigma'}^{\dagger} C_{j\sigma'} C_{i\sigma}. \tag{1}$$

Here, g is the nearest-neighbor attractive interaction constant and t is the transfer integral.

In order to consider the anisotropic superconductivity, we introduce following bond superconducting order parameter:

$$\Delta_{ij} = g\frac{1}{2}\langle C_{i\uparrow} C_{j\downarrow} - C_{i\downarrow} C_{j\uparrow}\rangle, \tag{2}$$

where we only consider the spin singlet pairing. For $d(\cos k_x - \cos k_y)$-wave superconductors, these bond order parameters become as

$$\Delta_{i,i+\hat{\mathbf{x}}} = \Delta = -\Delta_{i,i+\hat{\mathbf{y}}}, \tag{3}$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are lattice vectors along x- and y-axes, respectively.

Then the mean field Hamiltonian becomes as follows:

$$H_{\text{MF}} = -t \sum_{\langle ij\rangle\sigma}^{\text{n.n.}} \left(C_{i\sigma}^{\dagger} C_{j\sigma} + \text{h.c.}\right) - \sum_{\langle ij\rangle}^{\text{n.n.}} \left(\Delta_{ij}\left(C_{i\uparrow}^{\dagger} C_{j\downarrow}^{\dagger} - C_{i\downarrow}^{\dagger} C_{j\uparrow}^{\dagger}\right) + \Delta_{ij}^{*}\left(C_{i\uparrow} C_{j\downarrow} - C_{i\downarrow} C_{j\uparrow}\right)\right). \tag{4}$$

Then the Bogoliubov–de Gennes equation of this Hamiltonian for the quasi-particle wave functions u_i^n and v_i^n and their eigenvalues E_n becomes as

$$E_n u_i^n = -t \sum_{j}^{\text{n.n. of i}} u_j^n - \mu u_i^n - \sum_{j}^{\text{n.n. of i}} \Delta_{ij} v_j^n, \tag{5}$$

$$E_n v_i^n = t \sum_{j}^{\text{n.n. of i}} v_j^n + \mu v_i^n - \sum_{j}^{\text{n.n. of i}} \Delta_{ij} v_j^n. \tag{6}$$

The chemical potential μ is determined by the number conservation condition

$$N_e = 2\sum_n \left[f(E_n)\sum_i |u_i^n|^2 + (1 - f(E_n))\sum_i |v_i^n|^2\right]. \tag{7}$$

And the gap-equation becomes as

$$\Delta_{ij} = g\sum_n \left[u_i^n v_j^{n*}(f(E_n) - 1) + v_i^n u_j^{n*} f(E_n)\right]. \tag{8}$$

At the boundary we set the free boundary condition, because we consider finite systems. We solve these equations self-consistently.

Using the quasi-particle wave functions and the energy eigenvalues, we can calculate the local density of states as follows:

$$N(i,E) = -\sum_n \left[|u_i^n|^2 f'(E_n - E) + |v_i^n|^2 f'(E_n + E)\right]. \tag{9}$$

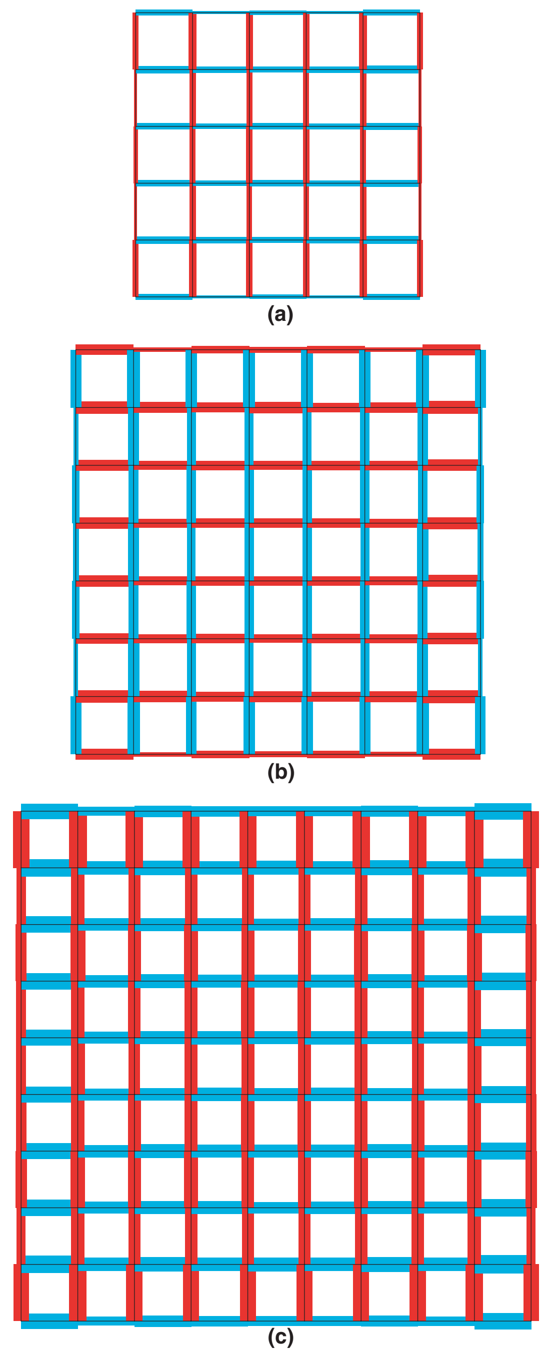

Fig. 1. Superconducting bond order for 6×6 (a), 8×8 (b) and 10×10 (c) lattices. Electron numbers are set to 32 for 6×6, 58 for 8×8 and 90 for 10×10.