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Enhancement of the critical current in quasiperiodic pinning arrays: One-dimensional chains and Penrose lattices

Vyacheslav R. Misko [a,b,c,*], Sergey Savel'ev [b,c], Franco Nori [b,d]

[a] Department of Physics, University of Antwerpen (CGB), B-2020 Antwerpen, Belgium
[b] Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama, 351-0198, Japan
[c] Center for Theoretical Physics, Center for the Study of Complex Systems, Department of Physics, University of Michigan, Ann Arbor, MI 48109-1040, USA
[d] Department of Physics, Loughborough University, Loughborough LE11 3TU, United Kingdom

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Abstract

Here we summarize results from our study of the critical depinning current J_c versus the applied magnetic flux Φ, for: (i) quasiperiodic (QP) one-dimensional (1D) chains and (ii) 2D arrays of pinning centers placed on the nodes of a five-fold Penrose lattice. In 1D QP chains, the peaks in $J_c(\Phi)$ are determined by a sequence of harmonics of the long and short segments of the chain. The critical current $J_c(\Phi)$ has a remarkable self-similarity. In 2D QP pinning arrays, we predict analytically and numerically the main features of $J_c(\Phi)$, and demonstrate that the Penrose lattice of pinning sites provides an enormous enhancement of $J_c(\Phi)$, even compared to triangular and random pinning site arrays. This huge increase in $J_c(\Phi)$ could be useful for applications.

Keywords: Vortex lattice; Critical current; Quasicrystals

1. Introduction

Recent progress in the fabrication of nanostructures has provided a wide variety of well-controlled vortex-confinement topologies, including different regular pinning arrays [1–5]. A main fundamental question in this field is how to drastically increase vortex pinning, and thus the critical current J_c, using artificially-produced periodic arrays of pinning sites (APS). The increase and, more generally, control of the critical current J_c in superconductors by its patterning (perforation) can be of practical importance for applications in microelectronic devices. A peak in the critical current $J_c(\Phi)$, for a given value of the magnetic flux per unit cell, say Φ_1, can be engineered using a superconducting sample with a periodic APS with a matching field $H_1 = \Phi_1/A$ (where A is the area of the pinning cell), corresponding to one trapped vortex per pinning site. However, this peak in $J_c(\Phi)$, while useful to obtain, *decreases very quickly for fluxes away from* Φ_1. Thus, the desired peak in $J_c(\Phi)$ is *too narrow* and not very robust against changes in Φ. It would be greatly desirable to have samples with APS with *many* periods. This multiple-period APS sample would provide either very many peaks or an extremely broad peak in $J_c(\Phi)$, as opposed to just one (narrow) main peak (and its harmonics). We achieve this goal (*a very broad* $J_c(\Phi)$) here by studying samples with *many built-in periods*. Here, we study vortex pinning by 1D quasiperiodic (QP) chains and by 2D APS located on the nodes of QP lattices (e.g., a five-fold Penrose lattice) [6]. We show that the use of the 2D QP (Penrose) lattice of pinning sites results in *a remarkable enhancement of* $J_c(\Phi)$, as compared to other APS, including triangular and random APS. In contrast to superconducting networks, for which only the areas of the network plaquettes play a role [7], for vortex pinning by QP pinning arrays, the specific geometry of the elements which form the QP lattice and their arrangement are important, making the problem far more complex.

* Corresponding author. Address: Department of Physics, University of Antwerpen (CGB), Condensed Matter Theory, Groenenborgerlann 171 2020 Antwerpen, Belgium. Tel.: +32 3 2653658; fax: +32 3 2653542.
E-mail addresses: vyacheslav.misko@ua.ac.be, vmisko@riken.jp (V.R. Misko).

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2. Simulation

We model a three-dimensional (3D) slab, infinitely long in the z-direction, by a 2D (in the xy-plane) simulation cell with periodic boundary conditions. The external magnetic field is applied along the z-direction. We perform simulated annealing simulations by numerically integrating the overdamped equations of motion (see, e.g., [8,9]):

$$\eta \mathbf{v}_i = \mathbf{f}_i = \mathbf{f}_i^{\mathrm{vv}} + \mathbf{f}_i^{\mathrm{vp}} + \mathbf{f}_i^{\mathrm{T}} + \mathbf{f}_i^{\mathrm{d}}. \tag{1}$$

Here $\mathbf{f}_i$ is the total force per unit length acting on vortex i, $\mathbf{f}_i^{\mathrm{vv}}$ and $\mathbf{f}_i^{\mathrm{vp}}$ are the forces due to vortex–vortex and vortex–pin interactions, respectively, $\mathbf{f}_i^{\mathrm{T}}$ is the thermal stochastic force, and $\mathbf{f}_i^{\mathrm{d}}$ is the driving force; η is the viscosity, which is set to unity. The force due to the vortex–vortex interaction is $\mathbf{f}_i^{\mathrm{vv}} = \Sigma_j^{N_{\mathrm{v}}} f_0 K_1(|\mathbf{r}_i - \mathbf{r}_j|/\lambda)\mathbf{r}_{ij}$, where N_{v} is the number of vortices, K_1 a modified Bessel function, λ the penetration depth, $\mathbf{r}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/|\mathbf{r}_i - \mathbf{r}_j|$, and $f_0 = \Phi_0^2/8\pi^2\lambda^3$. Here $\Phi_0 = hc/2e$. The pinning force is $\mathbf{f}_i^{\mathrm{vp}} = \Sigma_k^{N_{\mathrm{p}}} f_{\mathrm{p}} \cdot (|\mathbf{r}_i - \mathbf{r}_k^{(\mathrm{p})}|/r_{\mathrm{p}})\Theta \times [(r_{\mathrm{p}} - |\mathbf{r}_i - \mathbf{r}_k^{(\mathrm{p})}|)/\lambda]\mathbf{r}_{ik}^{(\mathrm{p})}$, where N_{p} is the number of pinning sites, f_{p} the maximum pinning force of each short-range parabolic potential well located at $\mathbf{r}_k^{(\mathrm{p})}$, r_{p} is the range of the pinning potential, Θ is the Heaviside step function, and $\mathbf{r}_{ik}^{(\mathrm{p})} = (\mathbf{r}_i - \mathbf{r}_k^{(\mathrm{p})})/|\mathbf{r}_i - \mathbf{r}_k^{(\mathrm{p})}|$. All the lengths (fields) are expressed in units of λ (Φ_0/λ^2). The ground state of a system of moving vortices is obtained by simulating the field-cooled experiments. For deep short-range (δ-like) potential wells, the energy required to depin vortices trapped by pinning sites is proportional to the number of pinned vortices, $N_{\mathrm{v}}^{(\mathrm{p})}$. Therefore, in this approximation, we can define the critical current as follows: $j_{\mathrm{c}}(\Phi) = j_0 N_{\mathrm{v}}^{(\mathrm{p})}(\Phi)/N_{\mathrm{v}}(\Phi)$, where j_0 is a constant, and study the dimensionless value $J_{\mathrm{c}} = j_{\mathrm{c}}/j_0$. We use narrow potential wells as pinning sites, with $r_{\mathrm{p}} = 0.04$–0.1λ.

3. Critical current in quasiperiodic arrays of pinning sites

3.1. 1D quasicrystal

A 1D QP chain [6] can be constructed by iteratively applying the Fibonacci rule ($L \to LS$, $S \to L$), which generates an infinite sequence of two line segments, long L and short S. For an infinite QP sequence, the ratio of the numbers of long to short elements is the golden mean $\tau = (1 + \sqrt{5})/2$ [6]. The position of the nth point where a new segment, either L or S, begins is determined (e.g., for $a_S = 1$ and $a_L = \tau$) by: $x_n = n + [n/\tau]/\tau$, where $[x]$ denotes the integer part of x. To study the critical depinning current J_{c} in 1D QP pinning chains, we place pinning sites to the points where the L or S elements of the QP sequence link to each other. The results of calculating $J_{\mathrm{c}}(N_{\mathrm{v}})$ for different chains and the same $\gamma = 1/\tau$ ($\gamma = a_S/a_L$ is the ratio of the length of the short segment, a_S, to the length of the long segment, a_L) are shown in Fig. 1. For sufficiently long chains, the positions of the main peaks in J_{c}, to a significant extent, do not depend on the length of the chain. The set of peaks in J_{c} includes a Fibonacci sequence: $N_{\mathrm{v}} = 13, 21, 34, 55, 89, 144$, as well as the peaks at first matching field (different for each chain) and other "harmonics". Being rescaled, normalized by the numbers

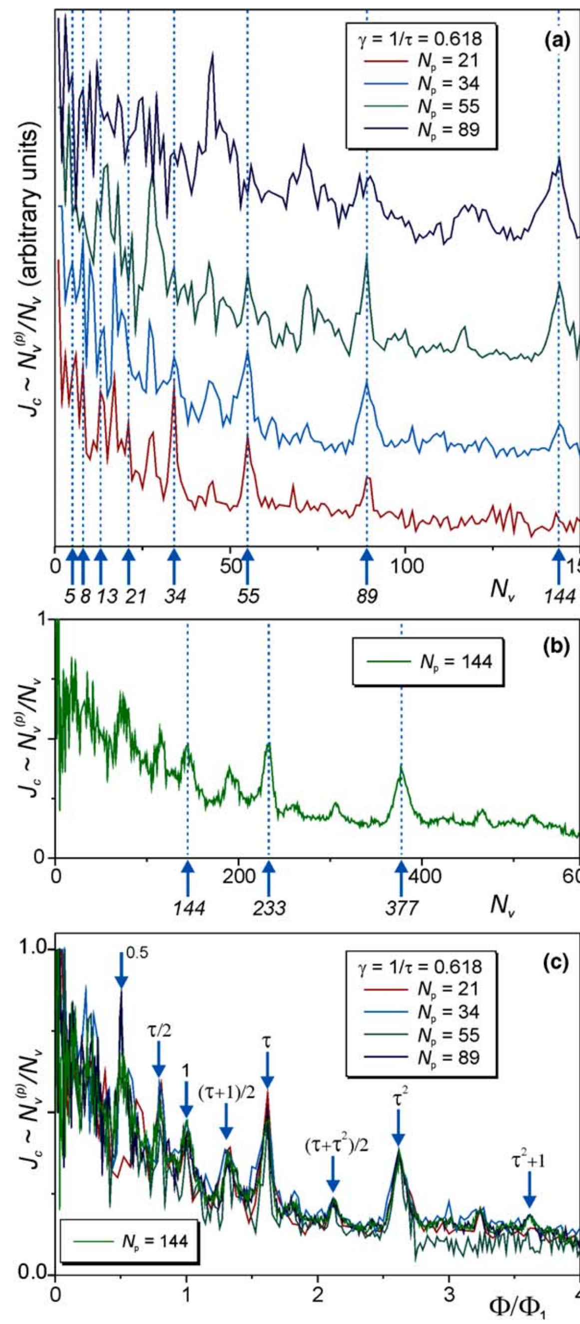

Fig. 1. (a) The critical depinning current J_{c}, versus the number of vortices, $N_{\mathrm{v}} \sim \Phi$, for 1D QP chains, $N_{\mathrm{p}} = 21$ (red bottom line), $N_{\mathrm{p}} = 34$ (blue line), $N_{\mathrm{p}} = 55$ (green line), and $N_{\mathrm{p}} = 89$ (dark blue top line), for $\gamma = a_S/a_L = 1/\tau$. Here we use: $f_{\mathrm{p}}/f_0 = 1.0$ and $r_{\mathrm{p}} = 0.1\lambda$. Independently of the length of the chain, the peaks include the sequence of successive Fibonacci numbers and their subharmonics. (b) $J_{\mathrm{c}}(N_{\mathrm{v}})$ for a long chain, $N_{\mathrm{p}} = 144$, and the same $\gamma = 1/\tau$. (c) The function $J_{\mathrm{c}}(\Phi/\Phi_1)$ for the same 1D chains (using same colors). The curves for different chains display the same set of peaks, namely, at $\Phi/\Phi_1 = 1$ (first matching field) and $\Phi/\Phi_1 = 0.5$, as well as at the golden-mean-related values: $\Phi/\Phi_1 = \tau$, $\tau/2$, $(\tau + 1)/2 = \tau^2/2$, $(\tau^2 + \tau)/2 = \tau^3/2$, $\tau^2 = \tau + 1$, $\tau^2 + 1$. This behavior demonstrates the self-similarity of $J_{\mathrm{c}}(\Phi)$. (For interpretation of the references in color in this figure legend, the reader is referred to the web version of this article.)