Nuclear Instruments and Methods in Physics Research A 828 (2016) 132–144

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

An analytical approach for beam loading compensation and excitation of maximum cavity field gradient in a coupled cavity-waveguide system

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ARTICLE INFO

Article history:
Received 27 September 2015
Received in revised form
8 April 2016
Accepted 6 May 2016
Available online 10 May 2016

Keywords:
High current accelerators
Beam cavity interaction
Coupled cavity-waveguide system
Beam loading compensation

ABSTRACT

The critical process of beam loading compensation in high intensity accelerators brings under control the undesired effect of the beam induced fields to the accelerating structures. A new analytical approach for optimizing standing wave accelerating structures is found which is hugely fast and agrees very well with simulations. A perturbative analysis of cavity and waveguide excitation based on the Bethe theorem and normal mode expansion is developed to compensate the beam loading effect and excite the maximum field gradient in the cavity. The method provides the optimum values for the coupling factor and the cavity detuning. While the approach is very accurate and agrees well with simulation software, it massively shortens the calculation time compared with the simulation software.

1. Introduction

In modern particle accelerators, radio frequency fields ranging from a few hundred to hundreds of megavolts per meter and from some kHz to many GHz are required. Such rf fields can be generated, for example, by klystrons, or semiconductor devices and then transferred through waveguides or coaxial lines to an accelerating structure composed of one or several rf coupled cavities. When a bunch of charged particles passes through an accelerating structure, it will be affected by the rf fields produced by the generator and at the same time it can induce its own electromagnetic fields to the structure. These additional rf fields are important for the designing and optimization of the accelerating structures and also klystrons. In case of accelerating structures without a careful design sometimes the beam induced field becomes comparable or even higher than the fields exerted by the generator. Moreover, since any bunch in the beam can see a fraction of its own induced wake fields and also the fields induced by the previous bunches, then minimization of these additional field is essential to avoid beam instabilities. This minimization process which is well known as the beam loading compensation was investigated by Wilson [1–3]. Although the approach is very simple and interesting, but the accuracy is not sufficient. Therefore, we attempt here to find a more accurate and comprehensive approach which can bring under control this undesired effect and also excite the maximum possible field gradient in the structure. In practice one resorts to the particle in cell simulation software like CST microwave studio [4] to find the optimum values for the coupling factor and cavity detuning which maximize the field gradient in the cavity. This requires a cumbersome trial and error procedure which would be massively time consuming. The main objective of the present paper is to find a fast and accurate approach to acquire these optimum values analytically. Our analytical approach is based on the Bethe theorem [5] for the equivalent dipole moments of the apertures separating a waveguide and a cavity and the normal mode expansion of the excited fields in the coupled cavity-waveguide system.

The approach also allows us to calculate the power which is extracted by a coupled cavity-waveguide system from a beam passing the structure. It can also be employed for designing and optimization of the output cavities of klystrons to have a maximum extracted power from the beam passing through the klystrons. In addition, for more convenience we attempt to find a relation between the coupling factor and the polarizability of the aperture [6]. This would allow us to find the aperture geometry parameters for any suggested value of the coupling factor. Since designing of an rf accelerator without any detailed studies of its rf components is impossible, therefore, we first consider the problem of rf fields excitation in the main components of an rf accelerating structure i.e. waveguides and cavities. Then in the next section we consider the problem of the waveguide excitation. Here we

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http://dx.doi.org/10.1016/j.nima.2016.05.019
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attempt to calculate the amplitudes of the excited modes in the waveguides in terms of the source's densities within the waveguide and the equivalent dipole moments of the apertures on its wall. In the third section, we describe a theoretical perturbation approach to calculate the excited field amplitudes inside a coupled cavity-waveguide system. Calculations in this section lead us to characterize a cavity-waveguide coupling system and find the excited field amplitudes produced only by the generator in terms of the system parameters. Later in the fourth section, we consider the whole system of a coupled cavity-waveguide system in the presence of a bunched beam. We make use of the calculations of the Sections 2 and 3 to find the excited field amplitudes due to the generator and beam together. These calculations are summarized at the end of this section in some self-consistent equations which can be applied for any kind of problems related to the accelerating structures and specially designing of standing wave accelerating structure with the goal of the beam loading compensation and maximum field gradient excitation. We finally present a few examples in Section 5 to illustrate the application of the calculations and also to make a comparison with the simulation results. We conclude the paper with a short discussion on the approach presented here and the obtained results in the conclusion. To have a better understanding about the electromagnetic modes excited in the cavities and waveguides, we also present brief derivations of the modes and their relations at the end of the paper in three appendixes.

2. Waveguide excitation

In Fig. 1 we have shown a two dimensional view of a cylindrical semi-infinite waveguide which is closed in one end except a coupling aperture Δa that is opened to the outer space. The waveguide can be connected through this aperture to any other waveguides, cavities or even to the free space. For this waveguide we consider a volume V (hatched area) which includes all the electromagnetic sources (indicated by ρ and $\vec{J}$) and also the aperture. This volume is enclosed by a surface S which is composed of three sub-surfaces S_1, S_2 and S_3. In this section our goal is to find the electromagnetic fields which can be excited in this volume and propagate along the waveguide in terms of the source densities and the equivalent dipole moment of the aperture. The approach is based on the Bethe theory [5] and normal modes expansion of the excited electromagnetic fields within the waveguide in frequency domain (denoted by $\vec{E}$ and $\vec{H}$) in terms of the forward and backward waveguide modes (denoted by $\vec{\vec{E}}_m^{\pm}$ and $\vec{\vec{H}}_m^{\pm}$). We assumed zero thickness for the walls separating the waveguide and its environment. The effect of small thicknesses of the separating wall on the excited fields can be found in Ref. [7]. However, according to expressions (B15 and B16, see Appendix B), the excited electromagnetic fields within the waveguide in frequency

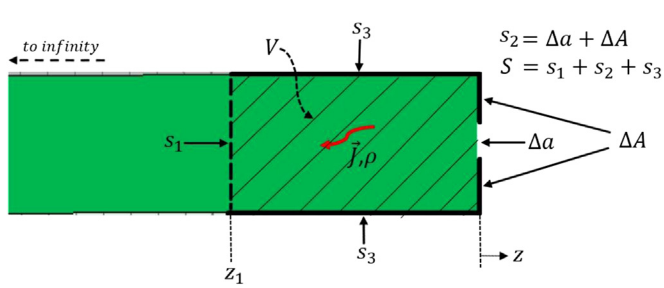

Fig. 1. Schematic diagram for a two-dimensional view of a cylindrical semi-infinite waveguide which is closed in one end where it has a coupling aperture Δa opened to its environment media. The marked region shows the volume V which includes all the electromagnetic sources and the aperture.

domain far from the locations of the sources and aperture can be expanded in terms of the waveguide modes as

$$\vec{E} = \sum_m c_m^+ \vec{\vec{E}}_m^+ + c_m^- \vec{\vec{E}}_m^- \tag{1}$$

$$\vec{H} = \sum_m c_m^+ \vec{\vec{H}}_m^+ + c_m^- \vec{\vec{H}} m \tag{2}$$

where $\vec{E}$ and $\vec{H}$ are the excited electromagnetic fields in the waveguide in frequency domain (for the relations between frequency domain and time domain see Appendix A). In addition, $\vec{\vec{E}}_m^{\pm}$ and $\vec{\vec{H}}_m^{\pm}$ are the mth electric and magnetic modes of the waveguide (see Appendix B), respectively. These modes are excited with the amplitudes $c_m^{\pm}$ and depending on the sign, they propagate in the positive or negative directions of the waveguide symmetry axis z. For calculation of the excited electromagnetic fields inside the waveguide it is enough to find the field coefficients $c_m^{\pm}$. To this end, we apply a perturbation method based on the Bethe theory [5] in which any aperture on the waveguide wall can be considered as a closed surface plus its equivalent dipole moments. We show the location of the center of the aperture with the position vector $\vec{\boldsymbol{r}}_a$ and its equivalent dipole moments in frequency domain by $\vec{m}$ and $\vec{p}$ for magnetic and electric, respectively. Thus for equivalent dipole moment densities of the aperture $\vec{M}$ and $\vec{P}$ in the frequency domain we can write

$$\vec{M}=\vec{m}\times\delta\left(\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}_a\right) \tag{3}$$

$$\vec{P}=\vec{p}\times\delta\left(\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}_a\right) \tag{4}$$

The strength of the dipole moments can be calculated from electrostatic and magnetostatic problems and it can be shown that [8]

$$[\vec{p}]^{(1)} = -[\vec{p}]^{(2)} = +\frac{\varepsilon_0\eta}{2}\left\{\left[\vec{E}_{\perp}(\vec{\boldsymbol{r}}_a\omega)\right]^{(2)} - \left[\vec{E}_{\perp}(\vec{\boldsymbol{r}}_a\omega)\right]^{(1)}\right\} \tag{5}$$

$$[\vec{m}]^{(1)} = -[\vec{m}]^{(2)} = +\eta\left\{\left[\vec{H}_{\parallel}(\vec{\boldsymbol{r}}_a\omega)\right]^{(1)} - \left[\vec{H}_{\parallel}(\vec{\boldsymbol{r}}_a\omega)\right]^{(2)}\right\} \tag{6}$$

where $[\vec{p}]^{(i)}$ and $[\vec{m}]^{(i)}$ indicate the strength of the dipole moments in frequency domain in view of the medium i and η is the polarizability of the aperture. The polarizability η of the aperture depends only on the geometry. As an example for a circular aperture with radius R_a, the related polarizability is equal to $\frac{8R_a^3}{3}$ [6]. In addition $\left[\vec{E}_{\perp}(\vec{\boldsymbol{r}}_a\omega)\right]^{(i)}$ and $\left[\vec{H}_{\parallel}(\vec{\boldsymbol{r}}_a\omega)\right]^{(i)}$ are the normal and parallel components of the electric and the magnetic fields, respectively. They should be looked at in the frequency domain and in view of the medium i at the center of the aperture. Sometimes one of the electric or magnetic dipole moments of the aperture is zero and we have only a magnetic coupling or an electric coupling problem. By means of the Maxwell equations (see Appendix A), for the excited electromagnetic fields, we can write the following useful identity

$$\vec{\nabla}\cdot\left(\vec{E}^*\times\vec{\vec{H}}_m^{\pm}+\vec{\vec{E}}_m^{\pm}\times\vec{H}^*\right) = -\left(\vec{\vec{E}}_m^{\pm}\cdot\vec{J}^*\right)+\mu_0 i\omega\vec{\vec{H}}_m^{\pm}\cdot\vec{M}^*+\left(i\omega\vec{\vec{E}}_m^{\pm}\cdot\vec{P}^*\right) \tag{7}$$

If we take an integral over the volume V and make use of the divergence theorem [9], expressions (5) and (6) and (B1) and (B2), we arrive at