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The *Spectrum Prize*: A simple algorithm to evaluate the relative sensitivity of γ-ray spectra, representative of detection systems

P. Spolaore

Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Legnaro, Legnaro (Padova), Italy

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ABSTRACT

A simple analysis of gamma spectra selected to represent the performance of different detection systems, or, for one same system, different operation modes or states of progress of the system development, allows to compare the relative average-sensitivities of the represented systems themselves, as operated in the selected cases. The obtained *SP* figure-of-merit takes into account and correlates the main parameters commonly used to estimate the performance of a system. An example of application is given.

1. Introduction

The great progress made in recent decades by the gamma-ray detector technology reflects the importance and usefulness of such apparatuses for many studies of fundamental and applied physics. The complexity of large Ge arrays is keeping increasing [1–5], so as to allocate more and more resources for the design, construction and operation activities.

In the process of setting up and optimizing a project, especially when this is complex and expensive, a trade-off among some desirable performances is usually unavoidable, so that it is important to evaluate step by step the possible compromise choices. In order to do this, significant parameters are used, in particular the detection efficiency ε, the energy resolution R and the ratio of signal to background P/B (peak-to-background) or P/T (peak-to-total) (e.g.: [6]). It is generally not possible to simultaneously maximize all these, and other possibly significant, parameters, for instance when filtering methods are used to reduce the radiation background with the consequence of cutting down the efficiency, or when budget or space limitations prevent to select a more effectual configuration. Therefore, the values of the relevant parameters have to be compared for different settings and their respective relevance for the targeted performance of the system must be determined, which is not always straightforward.

In fact, the aim to be achieved is, ultimately, the attainment of the best sensitivity, i.e. the capability of identifying and measuring the weakest possible signal, in the cases of interest. The sensitivity, however, is a parameter not as handy as those mentioned above (see Section 2). Due to this difficulty, various mathematical expressions have been proposed [7–10] to practically evaluate the sensitivity of detection systems, or correlated figures of merit, in particular for specific applications of interest. Other methods of estimating a set-up sensitivity make reference to the spectroscopic sensitivity for selected nuclear-structure high spin phenomena (e.g.: [11]), but this aspect is beyond the present discussion.

As, in the optimization process (which includes the stages of designing, construction and commissioning), the quality of the system is eventually evaluated by examining some gamma-spectra, obtained, with different detector settings, from computer simulations and/or experimental measurements, it could in fact be useful to have an indicator of the detection-system inclusive-performance, derivable from the acquired spectrum itself, without other details. Indeed, the quality of the detection system (which includes the detector, the using modalities of the radiation source and the data acquisition mode) is reflected in the quality of the produced spectra. Here and in the following, the expression “radiation source” refers, in general, to any kind of source: calibrated specimens, samples or in-beam generated nuclear reaction products.

The algorithm described in Section 2 provides a figure-of merit correlated with the system sensitivity and directly obtainable from the gamma-spectra analysis alone. This algorithm can be used to compare the quality of selected gamma spectra and, accordingly, the performance of associated detection systems. A simple case is used in Section 3 to show the practical application of the *Spectrum Prize* calculation.

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2. The *Spectrum Prize*

The sensitivity of a gamma detection system is defined as the smallest gamma activity that can be measured as a single γ-line, corresponding to a "peak" in a gamma spectrum. As the sensitivity depends on the γ-line energy, a reference energy value should be chosen when comparing the performance of different systems.

Considering a gamma spectrum, the sensitivity will be proportional to the area of the smallest peak (at the reference energy E_γ) which can be unambiguously identified as an effective signal (photopeak) in comparison with the underlying background fluctuation. The average background fluctuation, σ_B, would correspond to the "zero visibility" peak area. However, in practice, the minimum peak area "useful" for the γ-ray identification must be larger than σ_B, in order to assure that the possibly observed peak really corresponds to a gamma activity and is not due to statistical effects. A definition frequently used to practically identify the minimum detectable photopeak area (Minimum Detectable Amount-MDA) is provided by the "Currie equation" [7,12]:

$$\mathrm{MDA}_{Currie} = 4.653\ \sigma_B + 2.706\ ,\quad [\text{counts}]. \tag{1}$$

This MDA value (calculated for the reference energy E_γ) corresponds to the minimum number of counts in the selected peak for which a real activity can be assumed with a statistical probability of 95%. The corresponding activity, i.e. the sensitivity, is:

$$S(E_\gamma) = \mathrm{MDA}(E_\gamma)\ /\ [\epsilon_{\mathrm{abs}}(E_\gamma)\cdot f_\gamma \cdot t]\ ,\quad [\mathrm{Bq}], \tag{2}$$

with: $\varepsilon_{abs}(E_\gamma) = A_\gamma/(N\cdot f_\gamma \cdot t)$; A_γ=peak area; N=source disintegration rate; f_γ=γ-ray yield per disintegration, t=acquisition time.

It is obvious that for the calculation of the sensitivity the simple analysis of a gamma spectrum is not sufficient: a detailed knowledge, and possibly the ab-initio control, of some characteristics of the source, detector and acquisition modes are required.

A new quantity *SP* (*Spectrum Prize*) is here introduced, which is defined, for a gamma spectrum of a selected energy range including a set of selected γ-lines, as the ratio of the average peak area P_{av} to the area of the statistical fluctuation of the average underlying background B_{av}, as follows:

$$SP\ =\ P_{av}/\sqrt{B_{av}}; \tag{3}$$

$$P_{av} = P/n_P\ ;\quad P\ =\ \textstyle\sum_i P_i; \tag{4}$$

$$B_{av} = (B/n_{cT})\cdot(n_{cP}/n_P); \tag{5}$$

with: P_i=i-th peak area; n_P=number of peaks; B=total background; n_{cT}=number of channels of the analyzed spectrum; n_{cP}=sum of the widths, expressed as number of channels, of all the selected peaks.

Simplifying, *SP* will be expressed as:

$$SP\ =\ \left(\frac{P^2\ n_{cT}}{B\ n_P\ n_{cP}}\right)^{1/2}. \tag{6}$$

While comparing different spectra, *SP* provides a relative whole-spectrum rating which is, by definition, inversely proportional to the sensitivity referred to the defined "average peak". The average peak is not necessarily the minimum detectable one, nor any predefined multiple of it: the *SP* definition is very general, adaptable to any specific case on the basis of the selected spectrum range and γ-peaks.

While the *MDA* of the Currie equation (Eq. (1)) evaluates a minimum detectable peak-area, only based on the (local) background characteristics, and the sensitivity is an absolute value related to a particular gamma energy, *SP* is a relative parameter sensitive to the overall effectiveness of the spectrum for the purpose of obtaining the best sensitivity. *SP* could also be defined as a rating of the relative average sensitivity of the selected spectra, without forgetting that, contrary to the sensitivity, *SP* is "directly" proportional to the quality of the spectrum. *SP* is clearly intended to be used not as an absolute value, but for the comparison of selected spectra, chosen to represent different detection systems or different configurations of one same system.

Eq. (6) involves only quantities that can be immediately calculated by analyzing a gamma spectrum alone. This also allows to easily include the *SP* calculation in standard gamma spectra analysis codes.

The substantial significance of *SP* is illustrated by the relation of the factors of Eq. (6) with some commonly used quantities. Considering that the average-peak energy resolution R_{av} (=ratio of the average peak-width to the average peak-centroid, both here measured in channel numbers) is proportional (in a same-energy-range spectra comparison) to: $n_{cP}/(n_P\cdot n_{cT})$, that the absolute photopeak efficiency summed for the analyzed peaks is: ε_{abs}=$P/(Nft)$, with f= γ-ray yield per disintegration summed over the selected γ-lines, and defining G=Nft=total number of selected γ-rays emitted by the source during the spectrum acquisition, the following proportional relationship can be written:

$$SP\ \propto\ \left(\frac{P}{B}\frac{1}{R_{av}}\frac{\varepsilon_{abs}}{n_P}\frac{G}{n_P}\right)^{1/2}; \tag{7}$$

where ε_{abs} and G are related to the average peak by the division by n_P.

Of course, *SP* can be expressed with reference to the intrinsic, rather than absolute, efficiency by replacing G with G_{int}=total number of the selected γ-rays entering the detector. For instance, in case only the geometric solid angle has to be considered (no absorbing material interposed between the source and the detectors): G_{int}=$G\cdot\Omega/4\pi$.

Eq. (7) also shows the relationship among the parameters P/B, resolution, efficiency and acquisition statistics in determining the quality (in terms of sensitivity) of a gamma spectrum. Normally, one of such parameters can be used to evaluate the detection system if the others are equal; sometimes, combinations like: $(P/B)\cdot\varepsilon$; $(P/T)/(RE_\gamma)$; or $(P/B)\cdot\varepsilon/R(E_\gamma)$, and more, are used as useful figures of merit: Eq. (7) indicates that these comparisons are linear (in sensitivity, which is the overall performance parameter) if the square root of such parameters is used, instead of the simple values. This comes from the fact that the photopeak area is significant in relation to the background fluctuations rather than to the background itself.

When, in the compared systems, one or more of the quantities of Eq. (7) are known to be equal, *SP* provides a comparison of the remaining characteristics. More generally, when the effect of one (or more) of such quantities is not of interest for the spectra comparison and its values are different (and known) for all the spectra, *SP* can be corrected (re-normalized) for a same, selected, value of that quantity. For instance, in comparing spectra acquired with different statistics (namely with different acquisition times or different source activities) as represented by the factor *G*, *SP* can be re-normalized with reference to a selected statistics amount, equal for all the spectra. It is worth underlying that *SP*, being a sensitivity-proportional quantity, necessarily depends also on the acquired statistics. For completeness, it should be noted that the quantity *G* involves also the effect of the gamma multiplicity, which can be, in many cases, a significant (interesting) parameter.

Overall, *SP* measures the quality of the gamma spectrum in relation to its ability of revealing the γ-lines of lowest intensity, whatever the spectrum acquisition procedure has been. It is the responsibility and the opportunity of the investigator to select the spectra to be compared accordingly to the purpose of the comparison.