Nuclear Instruments and Methods in Physics Research A 808 (2016) 141–149

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

Analysis of intense beam instability in a general quadrupole focusing channel with image charge effect

A. Goswami, P. Sing Babu, V.S. Pandit

Variable Energy Cyclotron Centre, 1/AF, Bidhannagar, Kolkata 700064, India

ARTICLE INFO

Article history:
Received 4 September 2015
Received in revised form
2 November 2015
Accepted 11 November 2015
Available online 19 November 2015

Keywords:
Intense beam
Quadrupole focusing
Syncopated lattice
Image charge
Beam instability

ABSTRACT

The stability properties of transverse envelopes of mismatched intense continuous charge particle beam propagating in a general quadrupole focusing channel have been investigated in the presence of image charge effect due to a cylindrical conducting pipe. Phase shifts and growth factors of the envelope oscillations in the case of instability are calculated by numerical evaluation of the eigenvalues of linearly perturbed envelope equations for small deviations from the matched beam conditions. A detailed study on the region of instability and its dependence on the system parameters like occupancy of the quadrupole focusing field, syncopation factor, zero current phase advance, beam intensity etc. have been carried out. It has been found that the strength and regions of envelope instability due to the lattice and confluent resonances in the parametric space are affected by the presence of image charge.

1. Introduction

Recent years have shown a growing interest in the physics of advanced high current accelerators for applications such as heavy ion fusion, nuclear waste treatment, and spallation neutron source [1–3]. The most common accelerators used for these applications are linacs and storage rings. Periodic focusing systems are often used in such high power accelerators and beam transport systems to confine the intense beam within the available transverse apertures [4–7]. A critical factor in such applications is the requirement of low cost and high efficiency which needs proper optimization of accelerator and transport structures to minimize the beam particle loss due to emittance growth and halo formation [8–11]. Thus it becomes necessary to understand the collective behavior of space charge dominated beams by examining the beam envelope evolution and stability properties of mismatch perturbations around the matched beam envelope under the influence of both the beam space charge and image charge induced on the conducting walls of accelerator structures.

The linear stability properties of mismatched perturbations around the matched beam envelope have been studied by several authors [12–17]. Struckmeier and Reiser [15] have first analyzed the stability properties of beam envelope in uniform and periodic focusing channels by employing the linearized Kapchinskij–Vladimirskij (KV) envelope equations [18,19]. A more detailed study and review of such properties has been carried out by Lund and Bukh [16] for continuous focusing, periodic solenoidal, and periodic quadrupole transport channel assuming equal emittances for both transverse directions. In the above works, authors have neglected the effect of image charge. However, in most accelerators beam particles propagate through a conducting pipe and if the beam dimensions become comparable to the pipe dimensions, the image force from the pipe affects the beam dynamics. This is an unwanted effect and needs careful investigation to reduce its influence on the beam dynamics.

The dynamics and stability properties of space charge dominated beams in a periodic focusing lattice including the image charge effect have been investigated by several authors [20–23]. Their numerical results show that the first order image charge effect is almost negligible unless the beam is sufficiently eccentric and fills out a substantial portion of the beam pipe. An extended treatment with all higher order image charge effects [23] in a FODO lattice reveals that the effect of image charge is also negligible when the zero current phase advance is less than 90°. In a recent work [24] we have studied the stability properties of mismatched intense off axis beam only in a FODO lattice including the effect of image charge of a cylindrical conducting pipe. It is shown that for $\sigma_0 < 90^{o}$, the image-charge affects only the centroid motion and its effect on the matched beam envelopes is almost negligible. A noticeable effect on beam stability is observed when the beam is more off centered. The present work is an extension of the previous work and the main purpose is to examine the detailed stability properties of KV beam in a general quadrupole periodic channel with asymmetric drift distance between the focusing and defocusing quadrupoles (syncopated channel) and to

E-mail addresses: animesh@vecc.gov.in (A. Goswami), psb@vecc.gov.in (P. Sing Babu), pandit@vecc.gov.in (V.S. Pandit).

http://dx.doi.org/10.1016/j.nima.2015.11.056
0168-9002/

identify the different unstable modes of envelope perturbation with the system and beam space charge parameters including the effect of image charge.

The paper is organized as follows. In Sec. 2 the transverse envelope equations of intense charge particle beam with image charge effect due to conducting pipe are introduced. Equations of motion are obtained for small perturbations around the matched envelopes in a periodic focusing channel and procedure is discussed to analyze the envelope oscillation modes. In Sec. 3, a more rigorous analysis of beam instability in the presence of image charge effect in FODO channel as well as in syncopated channel by varying the quadrupole occupancy factor is discussed. Finally, conclusions are drawn in Sec. 4.

2. Theory

Consider the propagation of an intense, continuous charged particle beam with an average axial momentum $P=m\gamma\beta c$ inside a perfectly conducting cylindrical pipe of radius r_p, where β and γ are the usual relativistic parameters, m is the mass of the particle and c is the speed of light in vacuum. As customary in accelerator physics, we use $s=z=\beta ct$ as the axial coordinate measuring the distance along the beam axis and x–y plane as the transverse plane for the particle beam. Let $r_x(s)$ and $r_y(s)$ be the semi-axes of the transverse beam cross section in x and y planes, respectively, at location s. In the present analysis, it is assumed that both pipe and focusing fields are perfectly aligned with s axis. The energy spread in the beam is negligible and the transverse velocities of beam particles are very small as compared to the average axial velocity (i.e. $\dot{x}$, $\dot{y} << v=\beta c$) of particles.

Fig. 1 shows the schematic view of the transverse cross-section of the beam in a cylindrical pipe of radius r_p. We assume that the beam has uniform charge density (KV beam) within the elliptical transverse cross section with principal radii r_x and r_y aligned with the x and y coordinate axes.

2.1. Envelope model with image charge

The linear equations of transverse motion for a single particle of rest mass m and charge q in the applied field and beam self-field can be expressed as:

$$x''+k_x^2x=-\frac{q}{m\gamma^3\beta^2c^2}\frac{\partial\psi^S}{\partial x} \tag{1a}$$

$$y''+k_y^2y=-\frac{q}{m\gamma^3\beta^2c^2}\frac{\partial\psi^S}{\partial y} \tag{1b}$$

where prime denotes the derivative with respect to s. Here ψ^S is the self-field potential generated by the beam space charge

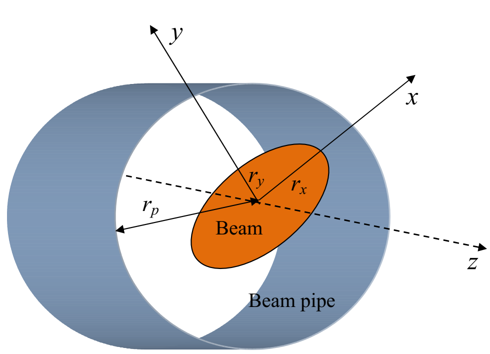

Fig. 1. Schematic of the transverse elliptical cross-section of an intense beam in a cylindrical conducting pipe.

including image charge effect due to the cylindrical conducting pipe. The corresponding electric field in x and y directions are, therefore, $E_x=-\partial\psi^S/\partial x$, $E_y=-\partial\psi^S/\partial y$. Here $k_x^2(s)$ and $k_y^2(s)$ are the linear applied focusing functions in x and y directions. The self field potential $\psi^S(x,y,s)$ in terms of the number density of beam particles $n(x,y,s)$ can be obtained by the solution to the Poisson's equation

$$\nabla^2\psi^S=-\frac{q\ n}{\varepsilon_0} \tag{2}$$

with the condition $\psi^S=\text{const.}$ on the perfectly conducting cylindrical wall located at radius $r=r_p$. Here ε_0 is the permittivity of free space.

The density profile of uniformly distributed beam particles with elliptical cross-section in this system can be expressed as

$$n(x,y,s)=\frac{n_0}{\pi\, r_x\ r_y}\Theta\left(1-\frac{x^2}{{r_x}^2}-\frac{y^2}{{r_y}^2}\right) \tag{3}$$

where $\Theta(\chi)=1$ if $\chi>0$ and $\Theta(\chi)=0$ if $\chi<0$ and r_x and r_y are the beam sizes along the transverse coordinate axes connected to the statistical moments of the particle distribution by

$$r_x=2\,\langle x^2\rangle^{1/2},\quad r_y=2\langle y^2\rangle^{1/2}\ \cdot \tag{4}$$

Here $\langle\cdots\rangle$ represents the ensemble average over the distribution.

The leading order direct field $\mathbf{E}^d$ and image field $\mathbf{E}^i$ produced by the uniform density beam defined in Eq. (3) in the conducting pipe of circular cross-section (Fig. 1) are given by [25–27]

$$E_x^d=\frac{\lambda}{\pi\varepsilon_0}\frac{x}{r_x(r_x+r_y)},\quad E_y^d=\frac{\lambda}{\pi\varepsilon_0\ r_y(r_x+r_y)}, \tag{5}$$

$$E_x^i=\frac{\lambda\ (r_x^2-r_y^2)}{8\pi\varepsilon_0r_p^4}x,\quad E_y^i=\frac{-\lambda(r_x^2-r_y^2)}{8\pi\varepsilon_0r_p^4}y\ \cdot \tag{6}$$

Here, $\lambda=q\int n\,dx\,dy$ is the constant line-charge density of the beam. Using Eqs. (5) and (6) into Eq. (1), a set of coupled transverse beam envelope equations can be derived by using second spatial moments of the beam distribution as:

$$r''_x+k_x^2r_x-\frac{2Q}{r_x+r_y}-\frac{\varepsilon_x^2}{r_x^3}=\frac{Q}{4r_p^4}(r_x^2-r_y^2)r_x \tag{7a}$$

$$r''_y+k_y^2r_y-\frac{2Q}{r_x+r_y}-\frac{\varepsilon_y^2}{r_y^3}=-\frac{Q}{4r_p^4}(r_x^2-r_y^2)r_y \tag{7b}$$

The term $Q=q\lambda/(2\pi\varepsilon_0mc^2\beta^2\gamma^3)=\text{const.}$, is the dimensionless beam perveance and ε_x and ε_y are the edge emittances of the beam in x and y planes defined by

$$\varepsilon_x=4\left[\langle x^2\rangle\langle x'^2\rangle-\langle xx'\rangle^2\right]^{1/2},\quad \varepsilon_y=4\left[\langle y^2\rangle\langle y'^2\rangle-\langle yy'\rangle^2\right]^{1/2} \tag{8}$$

These equations are similar to the standard KV envelope equations with an additional term on right hand side of both equations accounting for the dominant image charge effect due to the presence of conducting pipe. We like to point out here that similar equations have been derived by Lee, Close and Smith with higher order terms in Ref. 26. In the present analysis, we have assumed the emittance of the beam to remain constant which may increase due to the non-linear image charge effect. However, the increase in the emittance will be a minor effect for the space charge dominated beams.

2.2. Mismatched mode with image charge

In order to study the effect of image charge on the behavior of beam envelope we closely follow the procedure outlined in Ref. 15 and analyze the envelope oscillation modes to find the instability if any, for small deviations from the matched beam conditions. We