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Sifting through the remnants of heavy-ion collisions for observables sensitive to the nuclear equation of state

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ABSTRACT

Heavy-ion collisions provide an important probe of the nuclear equation of state. However, the dependence of different observables on the underlying interaction is not always clearly defined. A multidimensional analysis technique has been used to discriminate the observables sensitive to the asymmetry energy based on the results from Constrained Molecular Dynamics and a Stochastic Mean Field model. This multidimensional technique can be used to enhance our analysis of experimental observables and, thus, improve our ability to constrain the nuclear equation of state.

1. Introduction

As experimental data sets in physics become larger and more complicated, there is an increasing need for data analysis methods that can treat the data efficiently and in an unbiased manner. This is most apparent as we refine our understanding and focus our attention on finer details. In such cases, examining several observables simultaneously can lead to a consistent physical picture, where examining a single observable leaves room for uncertainty. The Sliced Inverse Regression Method (SIR) offers the ability to efficiently, and in an unbiased way, perform such analysis. In this paper, we demonstrate how the SIR method may be applied to a currently relevant topic in nuclear physics, namely constraining the asymmetry energy in the nuclear equation of state.

The nuclear Equation of State (EoS) for asymmetric nuclear matter describes the thermodynamic properties of nuclei and neutron stars, spanning a wide range in densities, temperatures, and isospin asymmetries [1–5]. The nuclear EoS and the density dependence of the asymmetry energy are of high interest, specifically as we gain access to rare, exotic beams and have greater access to probe the N/Z degree of freedom. The density dependence of the asymmetry energy (E_{sym}), commonly referred to as the symmetry energy [6], describes how the nuclear EoS depends on the asymmetry, $(N-Z)/(N+Z)$, of nuclear matter. Several theoretical models have been used to describe this density dependence of the asymmetry energy, utilizing microscopic many-body, mean field and quantum mechanical approaches [7–10]. This density dependence is important for describing astrophysical processes such as the creation of neutron stars and the dynamical collapse of supernovae [11,12], in addition to nuclear physics problems including understanding the structure of rare, exotic nuclei [13,14] and heavy ion collision dynamics.

There are a number of observables that can be analyzed when studying the EoS using heavy-ion collisions. Of course, some observables are more sensitive than others. A number of constraints have been placed on the EoS, both at supra- and subsaturation density, from a wide range of heavy-ion collision observables [15–23], neutron skin thicknesses [13,24–26], and astrophysical measurements [1–5,27]. Although there has been significant progress in recent years [15,28–30], tighter and more robust constraints should be obtained by examining multiple

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observables for the asymmetry energy within a single experimental data set. As larger and more complex data sets become increasingly common, systematic analysis of large multidimensional data sets is being discussed more often within the scientific community. In this paper, we explore the use of multiple observables simultaneously to extract a single composite observable that is more sensitive to the underlying interaction. Specifically, we have chosen the density dependence of E_{sym} as a case study for the purposes of this manuscript.

We propose a novel approach using a combination of several observables utilizing a "bottom-up" approach where one measures a number of observables, event by event, with the intent to increase the sensitivity to theoretical differences, such as the underlying nucleon–nucleon (NN) interaction of heavy-ion collisions. In this way, a larger weight can be given to the observables that differ more between the different NN interactions. The approach is based on a modern version of a classical approach [31] to multi-variate statistical analysis, namely using the Sliced Inverse Regression algorithm [32,33] within the R statistical environment [34].

We will show that the SIR method is able to do precisely as we propose. For the purpose of demonstrating the SIR method, we examine the dynamical break-up of the projectile- and target-like fragments from low energy heavy-ion collisions. Specifically, we are interested in the probability of a heavy ($Z \geq 3$) 3- or 4-body breaking of the reacting system at low-intermediate energy ($\sim$15 MeV/nucleon). In this case, the isospin content, alignment and velocity distributions of the excited primary fragments, quadrupole and octupole moments of the heaviest fragments, and observables associated with the emission of fragments from the low-density neck region produced in mid-peripheral heavy ion collisions have been proposed by theory to represent observables sensitive to the EoS [19,35–40]. We will also demonstrate the power of the SIR method by examining several observables associated with these break-ups and observe how the generation of a composite variable can increase the sensitivity.

2. Simulations

The interaction of ^{124}Sn+^{64}Ni at 15 MeV/nucleon was simulated with a Boltzmann–Nordheim–Vlasov based Stochastic Mean Field approximation (SMF), employing the test particle method [10,40], and the Constrained Molecular Dynamics (CoMD) [8] model. Three different parameterizations of E_{sym} were simulated for both SMF and CoMD. The density dependence of the asymmetry energy parameterizations is shown in Fig. 1. The asymmetry energy, generally, can be described in two ways. In the case where E_{sym} increases monotonically with increasing density (ρ), the relationship between E_{sym} and ρ can be described as "stiff", or asy-stiff (Fig. 1). Conversely the "soft", or asy-soft, can be generally described as the case when the relationship E_{sym} vs ρ increases to approximately saturation density ($\rho_0 \sim 0.16$ fm^{-3}) [41] and then begins to decrease above ρ_0. In the case of CoMD, the "softer" of the three parameterizations of E_{sym} does continue to increase monotonically with density. The stiff E_{sym} for both CoMD and SMF, in Fig. 1, lay on top of each other. For the super-stiff E_{sym}, although the lines lay close to each other, the super-stiff parameterization for CoMD is slightly more stiff at $\rho/\rho_0 > 1$. However, in the case of the asy-soft, E_{sym} for SMF is clearly much softer than for the CoMD. At saturation density ($\rho/\rho_0 = 1$), the E_{sym} curves intersect as this represents the well constrained normal nuclear density described by the semi-empirical mass formula for nuclei. At the energy of the simulated reactions for this analysis (15 MeV/nucleon), the 2-body separation (binary) that we are observing will be below saturation density due to the increase in excitation and a subsequent increase in volume just after reseparation. In Fig. 2, the density profile for a 2-body separation of the projectile-like and target-like fragments (PLF and TLF respectively) is depicted for ^{124}Sn+^{64}Ni at 15 MeV/nucleon at t=450 fm/c for an impact parameter of 7 fm in the reaction plane as simulated using the stiff E_{sym} from SMF. In Fig. 2, the Z-coordinate is parallel to the beam direction with the projectile traveling from low to high value on the Z-coordinate. The transverse-coordinate represents the plane transverse the beam direction. The PLF is depicted by the larger, deformed fragment on the right-hand side of the reaction plane. The strong deformation of the PLF and TLF, illustrated in Fig. 2, is predicted to be dependent on the stiffness of the E_{sym} [19,35–40].

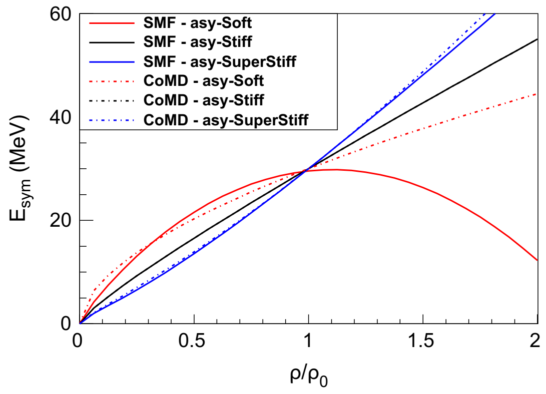

Fig. 1. Density dependence of the asymmetry energy for SMF and CoMD. Solid lines represent SMF, dashed lines represent CoMD with the soft parameterization in red (light gray), stiff in black and super-stiff in blue (dark gray). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

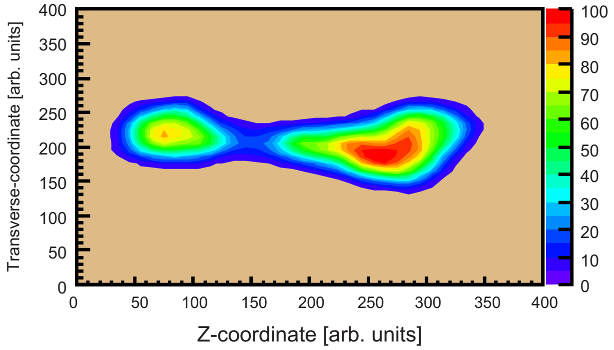

Fig. 2. A typical density contour of ^{124}Sn+^{64}Ni at 15 MeV/nucleon in the reaction plane at t=450 fm/c at an impact parameter of 7 fm in coordinate space as simulated by the stiff E_{sym} in SMF with 40 test particles per nucleon. The projectile-like fragment (PLF) is depicted by the larger, deformed fragment on the right and the target-like fragment (TLF) on the left. Density contours range from low to high represented by purple (darker gray) the edges of the profile to deep red (transitioning from dark to lighter grays) at the center of the profiles. The Z-coordinate is parallel to the beam direction with the projectile traveling from low to high value on the Z-coordinate. The transverse represents the plane transverse the beam direction. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Approximately 3000 and 500,000 events were simulated with each model, SMF and CoMD respectively, to train the SIR algorithm in identifying the way observables behave relative to a change in E_{sym}. The output of SMF (flat impact parameter distribution from 6 to 8 fm) was then treated with a coalescence code [42] to identify the free nucleons and clusters that appear in the exit channel based on the locations of the test particles in phase space. The coalescence code output was then filtered in order to select only on the PLF at t=450 fm/c. In the case of CoMD, a triangular