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Optimal performance of charged particle telescopes in space

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ABSTRACT

A Bayesian probabilistic data analysis method for energetic proton and ion data from charged particle telescopes in space is described. The telescope is assumed to consist of only a series of planar silicon detectors with graduated thicknesses. The method is based on a range-straggling function and makes optimal use of energy loss measurements in each detector. It provides accurate incidence angle estimates for particles stopping in the telescope, allowing accurate element identification and possible isotope identification. It also provides energy estimates for high-energy particles going through the telescope without stopping. Examples are shown for simulated telescope design performance tests and application to real space-particle data.

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1. Introduction

Charged particle telescopes, consisting of a series of aligned planar silicon detectors, have been used frequently in space applications, because of their low mass and power requirements and ability to make accurate energy measurements [1–4]. For protons the useful kinetic energy range is $\sim$10 MeV–1 GeV. Data analysis using the $\Delta E - E'$ technique provides element discrimination for particles stopping in the telescope [1,5]. Isotope discrimination is also possible when a tracking system for determining incidence angle is included [2]. However, such a system is complex and often unavailable, so it is of interest to evaluate optimal performance without one. (In this context, "optimal" refers to the most accurate possible results for a given telescope design, accounting for all measured data.) An optimal method of energy estimation for high-energy particles, that go through the telescope without stopping, is also of interest.

Two main problems must be addressed in any data analysis scheme: (i) path length variations caused by particles incident on the telescope at differing angles, and (ii) range straggling, which causes fluctuations in the energy loss of particles with known incident energy in a known thickness of silicon. To address each of these, a Bayesian probabilistic data analysis method is described, based on a theoretical straggling function, that makes optimal use of all energy loss measurements. Small-angle nuclear scattering is neglected and it is assumed that there is no passive absorbing material between detectors. Apart from these limitations, the method is of general applicability to proton and heavy-ion data.

2. Probabilistic formulation

The straggling function $f(\Delta, E, x)$ is the normalized probability density function (PDF) for energy loss Δ in a thickness x of silicon from an initial energy E. For a particle entering a telescope of N detectors, with thicknesses $\{x_i, i=1\ldots N\}$, the energy losses in consecutive detectors are independent random processes. Therefore, the joint conditional PDF for a set of energy losses $\{\Delta_i, i=1\ldots n\}$ with $n \le N$, given incident energy E and angle θ, is the product of the individual straggling functions accounting for energy loss in previous detectors:

$$f_n(E,\theta) = \prod_{i=1}^{n} f(\Delta_i, E-E_{i-1}, x_i \sec\theta) \tag{1}$$

where the total energy loss up to detector i is

$$E_i = \sum_{j=1}^{i} \Delta_j \tag{2}$$

and $x = x_i \sec\theta$ is the path length in detector i. If there is no prior knowledge of E or θ then, from Bayes' theorem, $f_n(E,\theta)$ is also the unnormalized joint conditional PDF for E and θ given a set of measured Δ_i (the set is referred to as an "event").

If energy losses are recorded in detectors 1 through k with $k<N$, so that the particle stopped in detector k, then $E=E_k$, the sum of all energy losses. Straggling in detector k is irrelevant because the particle lost all of its remaining energy there, and therefore $n=k-1$. An estimate of θ is the expectation, or mean value:

$$\bar{\theta} = \frac{1}{A_k}\int \theta f_{k-1}(E_k,\theta)\, d\theta \tag{3}$$

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where

$$A_k = \int f_{k-1}(E_k, \theta)\, d\theta \tag{4}$$

Depending on the design of the telescope, it may be possible for a particle to trigger detector k and then exit the telescope without triggering detector $k+1$. Then it is unclear whether it actually stopped in detector k. In such cases, it is necessary to determine whether the total energy loss E_k is consistent with a stopping particle and to discard events for which it is not. This is similar to the case in which all N detectors are triggered.

If losses are recorded in all N detectors then the particle may have stopped in detector N or it may have gone through. The joint PDF for E and θ, accounting for both possibilities, is $\delta(E-E_N)f_{N-1}(E_N,\theta)+f_N(E,\theta)$. Estimates of E and θ are

$$\overline{E} = \frac{1}{B_N}\left[E_N A_N + \iint E f_N(E,\theta)\, dE\, d\theta\right] \tag{5}$$

$$\overline{\theta} = \frac{1}{B_N}\left[\int \theta f_{N-1}(E_N,\theta)\, d\theta + \iint \theta f_N(E,\theta)\, dE\, d\theta\right] \tag{6}$$

where

$$B_N = A_N + \iint f_N(E,\theta)\, dE\, d\theta \tag{7}$$

The probability that the particle stopped in detector N is A_N/B_N. If it is unclear whether the particle entered through the front or back of the telescope then both possibilities can be similarly included.

If element or isotope identification is required then each possible particle species has its own straggling function based on its mass and charge. The corresponding probability densities should be combined to determine the most likely species. For example, if a particle stopped in detector k and there are two possible species with straggling functions f and g, then the combined density, replacing f_{k-1} in Eqs. (3) and (4), is $f_{k-1}+g_{k-1}$. The probability that the particle is of the f species is

$$P_f = \frac{\int f_{k-1}(E_k,\theta)\, d\theta}{\int [f_{k-1}(E_k,\theta)+g_{k-1}(E_k,\theta)]\, d\theta} \tag{8}$$

Other possible species can be added to the denominator.

3. Straggling function

The method just described is generally applicable if the straggling function is known. For fast particles through thin detectors, complex techniques are required to compute f, though it may be approximated by a Landau distribution [6]. For slower particles or moderate detector thicknesses, f is well represented by a Vavilov distribution [7,8]. For yet slower particles and/or thicker detectors, a Gaussian or distorted Gaussian approximation is adequate. The log-normal distribution of Chibani [9] is used here. It has the advantages of allowing analytic evaluation and rapid sampling for Monte Carlo simulations. It is considered valid for $0.3<\kappa<10$, where κ is the Vavilov parameter [9]. For $\kappa>10$ a Gaussian approximation is appropriate. Examples of particle energies corresponding to each of these cases are given in the following sections. In both, a mean energy loss $\overline{\Delta}$ must be computed separately, as follows.

The range $R(E)$ of charged particles in Si is related to stopping power dE/dx that, for all energies of interest, is well represented by the Bethe formula [8]. Here, proton range $R_p(E)$ is computed from a log-polynomial fit to a projected range table for protons in Si from NIST/PSTAR [10]. For heavier particles the range is well approximated by the scaling relationship $R(E)=R_p(E/M)M/Z^2$ [1], where M is the particle mass in proton mass units, and Z is the charge number. The mean energy loss in thickness x is then

$$\overline{\Delta} = E - R^{-1}(R(E)-x) \tag{9}$$

where the inverse range R^{-1} is the energy of a particle with the given range of its argument.

Sample straggling functions $f(\Delta,E,x)$ for protons in Si, with $x=1$ mm and various incident energies, are shown versus Δ in Fig. 1. Comparing to the Vavilov distribution, the log-normal approximation is seen to be reasonably accurate for $E \lesssim 50$ MeV in this case.

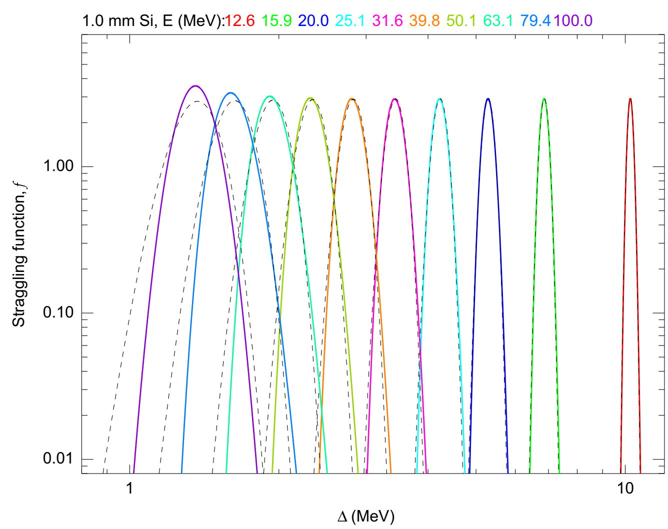

Fig. 1. Sample straggling functions f versus energy loss Δ for protons in 1 mm of Si, using the log-normal approximation [9] (color coded by initial energy E) and the Vavilov distribution [7] (dashed). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

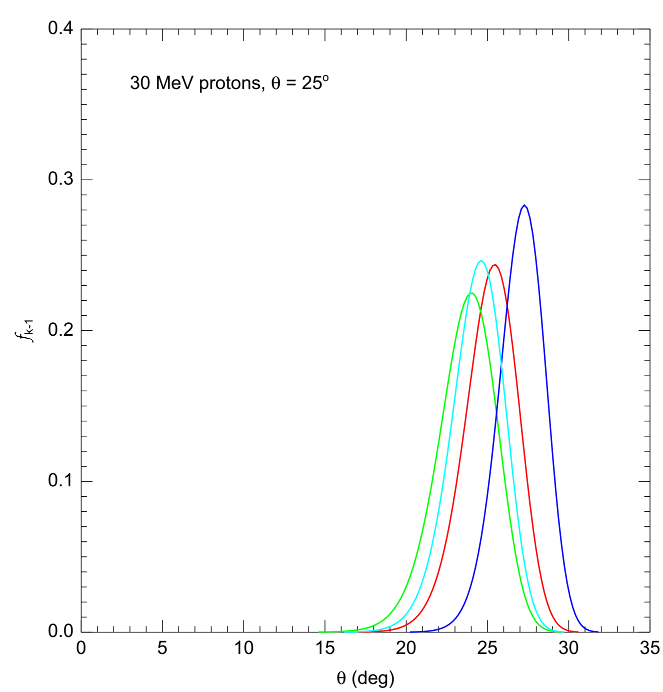

Fig. 2. Examples of simulated probability density f_{k-1} for incidence angle θ of stopping protons with a 5-detector telescope described in the text. Energy losses Δ_i were sampled from straggling functions with initial $E=30$ MeV incident at $\theta=25°$. Separate colors are used to distinguish each case. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)