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Fast decoding algorithms for coded aperture systems

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ABSTRACT

Fast decoding algorithms are described for a number of established coded aperture systems. The fast decoding algorithms for all these systems offer significant reductions in the number of calculations required when reconstructing images formed by a coded aperture system and hence require less computation time to produce the images. The algorithms may therefore be of use in applications that require fast image reconstruction, such as near real-time nuclear medicine and location of hazardous radioactive spillage. Experimental tests confirm the efficacy of the fast decoding techniques.

1. Introduction

Coded aperture imaging has become the major technique for forming images in the high energy domain [1–4]. This imaging method has been proposed and used in a number of applications, most notably high energy astronomy [4] and nuclear medicine [5], although its use has also been suggested for tracking radiation contamination [6] and flaw detection in mechanical structures [7].

In the coded aperture imaging technique, an aperture consisting of opaque and transparent elements is placed between a photon emitting source and a position sensitive detector. During observation, photons not absorbed by the opaque aperture elements pass through to the detector. The result is a shadowgram on the detector which needs to be subsequently decoded to produce a reconstructed image of the source distribution. Many patterns have been proposed for the aperture. The Fresnel zone plate [1] and the random pinhole aperture [2] have largely been superseded by patterns having perfect imaging capability, where cross-correlating the aperture with the decoding function gives a perfect delta function, although a random pattern has been used on the recent *SWIFT* mission [8]. Early examples of perfect apertures are the uniformly redundant arrays (URAs) [3] and the modified uniformly redundant arrays (MURAs) [9]. Discoveries of other perfect apertures were motivated by the desire to tackle certain problems, such as having antisymmetric apertures [10,11], self-supporting apertures [12,13] and apertures with low throughput [14,15]. Although perfect apertures are often seen as desirable, it should be noted that in a practical context other effects such as imperfections in the detector, variations in detector efficiencies and various mechanical constraints, such as artifacts in the shadowgram caused by the grid structure used to support the aperture elements, can dominate noise created by using imperfect apertures [16]. Therefore the choice of aperture is not always critical.

While the perfect apertures give good images with the image quality independent of the source distribution, cross-correlating the shadowgram with the decoding function is time consuming, particularly for systems with large numbers of aperture elements and detector pixels. Typically, if there are N elements in the unit pattern of an aperture and M pixels on a detector, decoding requires N^2M^2 multiplications, with the time required to perform this decoding being approximately proportional to the same quantity. For example the *IBIS* telescope on board the *INTEGRAL* project has $N=M=53$ [17,18] and the SAX-WFC device has $N=M\simeq 256$ [19]. Therefore the decoding time will increase very rapidly with system size. In applications where economy of time is important it therefore becomes desirable to find faster methods of reconstructing the images. Examples include near real-time imaging in nuclear medicine where fast diagnosis is required (similar studies in real time medical ultrasound imaging have been done by Heimdal et al. [20] and Choe [21]). Also speed may be required in the rapid location of radioactive sources, for example during a spillage of potentially dangerous material in order that the hazard may be dealt with urgently [6].

In addition to the aforementioned cases, there are a number of non-coded aperture applications and systems that use similar correlation techniques, and which may therefore also benefit from faster decoding. These would include applications requiring very large arrays, possible examples being channel estimation for antenna systems [22], time frequency coding [23], radar applications [24], communications [25], cryptography [26] and built-in tests for very large scale integration (VLSI) circuits [27,28].

In the case of coded aperture imaging, Roques [29] has described a fast decoding algorithm for the URAs, making use of the special properties of these systems and their decoding functions. When used, Roques' method gives a substantial time saving

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in the decoding of images produced using a URA. However, the main drawback is that URAs are limited in terms of the parameters available to the user, in particular those related to the above-mentioned desirable characteristics. Firstly URAs are not fully antisymmetric. Although mechanical attempts have been made to utilise the partial antisymmetry of URAs [30], implementing such methods in a space application may be hazardous. Secondly, all URAs have a throughput of approximately 50% and hence offer very little flexibility in terms of this particular parameter. Finally, because the URAs have such a high throughput, they do not offer much rigidity of support, unlike the self-supporting apertures. In this paper we describe fast decoding algorithms for some of the aperture patterns which possess the characteristics described. We investigate the following. Firstly we study the square MURA configurations [9], of which a subset is antisymmetric on 90° rotation [11]. Secondly we analyse those configurations described by the author that are created from products of individual one-dimensional coded aperture systems [14]. For ease of discussion, we refer to these as *product* apertures. The product apertures have low throughput and are all fully self-supporting, special cases of which are the pseudo noise product (PNP) arrays [12] and the M–P and M–M arrays [13].

2. Standard image reconstruction

In many cases, a coded aperture system based on a rectangular geometry is defined by a *unit pattern* of size $v \times w$ elements [3,9,14]. Exposure to the source typically generates a detector shadowgram, $P(i,j)$, where $0 \le i \le v-1$ and $0 \le j \le w-1$. The unit pattern of the decoding function is represented by $G(i,j)$ with $0 \le i \le v-1$ and $0 \le j \le w-1$. Often the full decoding function is represented by a $2v-1$ by $2w-1$ repetition of G [3,29], although in this paper we use here the modulo v and w form of the function G for ease of notation. The reconstructed image is given by

$$I(k,l) = \sum_{i=0}^{v-1} \sum_{j=0}^{w-1} P(i,j) G(i+k, j+l) \quad (1)$$

where $i+k$ is taken modulo v, $j+l$ is taken modulo w, $0 \le k \le v-1$ and $0 \le l \le w-1$. In the standard reconstruction method this double summation is completed in its entirety using a computing device. Because of the two summations over v and w terms, reconstruction of the function I therefore requires $v^2 w^2$ multiplications, which becomes very large for large systems.

In any fast reconstruction technique, we attempt to circumvent the necessity of calculating every term in either or both the summations in Eq. (1). This can be achieved by taking advantage of the special way the decoding function is created. In his paper Roques used the special properties of the URAs to describe a fast decoding algorithm for these systems. In the next two sections we describe fast decoding algorithms for the square MURAs and the product apertures, of which the PNP, M–P and M–M apertures are special cases.

3. Square MURA apertures

The MURA apertures were introduced by Gottesman and Fenimore [9]. In their paper they describe two different MURA types, in which they mosaic onto either a linear, hexagonal or a square configuration. We here demonstrate that images created using the square configuration MURAs can be decoded using a fast algorithm. The basic idea is very similar to that of the fast decoding algorithm of URAs [29]. Let p be an odd prime and define C_p as follows:

$$C_p(i) = \left(\frac{-1}{p}\right)\left(\frac{i}{p}\right) \quad (2)$$

where (i/p) is the Legendre symbol for p and $0 \le i \le p-1$. We now recall that for a square MURA of side p elements where p is an odd prime, the unit pattern of the decoding function G is given by the following:

$$G(i,j) = \begin{cases} -1 & \text{if } i=0, j \ne 0 \\ 1 & \text{if } j=0 \\ C_p(i)C_p(j) & \text{otherwise} \end{cases} \quad (3)$$

where $0 \le i,j \le p-1$. Substituting the various values of $G(i,j)$ from Eq. (3) into Eq. (1) and rearranging gives

$$I(k,l) = \sum_{\substack{i=0 \\ i \ne -k}}^{p-1} C_p(i+k) \sum_{\substack{j=0 \\ j \ne -l}}^{p-1} P(i,j) C_p(j+l) - \sum_{\substack{j=0 \\ j \ne -l}}^{p-1} P(-k,j) + \sum_{i=0}^{p-1} P(i,-l). \quad (4)$$

The resulting expression for I is virtually identical to that for the URAs as given by Roques [29], the only difference being the nature of the signs in the second and third terms of the right hand side of Eq. (4). The methods used by Roques are therefore directly applicable to the MURAs and so any time saving gained when using the MURAs is equivalent to that quantified by Roques [29].

It is interesting to note that Gottesman and Fenimore predicted the possible existence of a fast decoding algorithm for the square MURAs, due to the inherent symmetry of these systems [9, p. 4352]. A point that makes this development particularly important is the fact that a subset of the square MURAs has been shown to be antisymmetric on 90° rotation [11]. Antisymmetric apertures are useful in the removal of systematic spatial variation in the detector background [10,31]. Therefore with a slight modification, the method described above can be used and so there exists a type of antisymmetric aperture for which fast decoding can be employed.

4. Product apertures

The product apertures are synthesised using the products of single coded aperture systems, called *primitive* systems [14]. The PNP [12], M–P and M–M [13] apertures are all subsets of the product apertures. Although some primitive systems can have orders that are non-prime (for example a PN sequence can be of order 63), because of the methods used, we are concerned here with primitive systems that have prime orders. Let G be the decoding function of a product system, composed of n primitive systems of orders $p_1, p_2, \ldots, p_n$, where the p_x are all prime. We do not need to define the dimensionality of the coded aperture system being used, since the theory described below applies to all cases, including systems with more than two dimensions. Note, however, that while the p_x do not necessarily have to be distinct, if there are m dimensions to a product system, there are at most m values of p_x that can be equal, and only then if there is no more than one primitive system of order p_x used in any one dimension.

We define the set of n functions C_x by the following two equations:

$$C_x(0) = 1 \quad \text{for all } x \quad (5)$$

$$C_x(i) = \left(\frac{-1}{p_x}\right)\left(\frac{i}{p_x}\right) \quad \text{if } i \ne 0 \quad (6)$$

where $x = 1, 2, \ldots, n$. The decoding function of a product aperture is given by