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# A compound parabolic concentrator as an ultracold neutron spectrometer

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ABSTRACT

The design principles of nonimaging optics are applied to ultracold neutrons (UCN). In particular a vertical compound parabolic concentrator (CPC) that efficiently redirects UCN vertically into a bounded spatial volume where they have a maximum energy $mga$ that depends only on the initial phase space cross sectional area $\pi a^2$ creates a spectrometer which can be applied to neutron lifetime and gravitational quantum state experiments.



## 1. Introduction

Compound parabolic and elliptical concentrators, designed using the edge ray principle familiar from nonimaging optics [1], have been used with success to collimate, focus and concentrate cold neutrons onto a distant target using the approximation that the neutrons travel in a straight line, as is the case with light [2,3]. In the ultracold limit of the neutron, around a few hundred neV, these straight line reflectors suffer from chromatic aberration as the kinetic energy and gravitational potential approach equal magnitude. In previous uses, this chromatic aberration has been detrimental, particularly for grazing-angle, sideways pointing concentrators as only a narrow portion of the available phase space reaches the target, and the energy spectrum is heavily distorted. While UCN microscopes have been designed using imaging optics to eliminate chromatic aberration [4,5], to date nonimaging concentrators have not been used to measure UCN spectra because as the scale of the concentrator approaches the scale of the curvature of the flight paths, a standard concentrator on its side destroys the spectral information. In this paper we will show that certain cases of these concentrators can be designed for spectroscopy in the UCN limit if the gravitational curvature of the neutrons is taken into account. While previous gravitational spectrometers have achieved remarkable energy resolutions of peV [6], they have done so at the cost of UCN number efficiency by removing unused phase space volume or waiting for UCN to defuse into the measured state from a storage vessel. We show that the principles familiar from nonimaging optics can be applied to UCN optics to design an efficient vertical spectrometer, which utilizes much of this unused phase space on a first pass, in a time much shorter than the neutron lifetime, or more appropriately, the vessel storage time. As a prime example, we investigate a unique case of a vertical compound parabolic concentrator (CPC) which can quickly isolate UCN in bands as narrow as $\frac{1}{3}mga$ FWHM for a guide radius $a$. For 6 cm diameter guides, this gives 1 neV resolution, achievable after one pass through the optical system in the apogee time of the UCN, $v_0/g$. Such a CPC spectrometer can be used for a number of new experiments such as measuring the neutron lifetime using UCN.

## 2. Nonimaging UCN optics

In imaging neutron optics, each imaged neutron path is determined by Fermat's principle which also coincides with the classical action principle

$$\delta \int_a^b \frac{mv^2}{\hbar} dt = \delta \int_{t_a}^{t_b} L\, dt = 0, \tag{1}$$

so that the advancing wavefronts and the classical paths also coincide [7,8]. We may consider all potentials, gravitational, magnetic and the Fermi potential, as affecting UCN via an effective index of refraction

$$n^2(\vec{r}) = 1 - \frac{\lambda^2}{2m\hbar^2} V(\vec{r}). \tag{2}$$

For a rotationally symmetric system with angular momentum $\ell$, we have

$$V(r,z) = mgz \pm \mu_n B(r,z) + \frac{\ell^2}{2mr^2} + V_F(r,z). \tag{3}$$

For an imaging system, the integral from Fermat's principle is stationary for all neutron paths from object aperture to the image

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so we can solve for imaging optical surfaces using

$$\delta \int_a^b n^2\, dt = 0. \tag{4}$$

To design a nonimaging optical system, we relax the requirement that this is satisfied for all paths emanating from the input aperture. Not every point in the input aperture must have a conjugate point in the target space. Instead we rely on the edge ray principle familiar from nonimaging photon optics [1] that states that imaged paths serve only as the *boundary* for the phase space volume of all other enclosed paths. We use Fermat's principle only to solve for the classical edge paths and pick a reflector surface or potential geometry parameters that map the input aperture extrema to the extrema of the output region. All paths within that imaged path boundary will be guaranteed to arrive at the target region, regardless of the path taken, the number of reflections, or the path complexity.

## 3. Compound parabolic concentrators

From nonimaging optics for noncurvilinear rays, the general CPC family has reflective walls of a parabola that are tilted by the acceptance angle $\theta_A$ to the axis of rotation. The traditional CPC design [1] with an aperture at $z=0$ radius $a$, has the parametric form

$$r(\varphi) = \frac{2a' \sin(\varphi - \theta_A)}{1-\cos\varphi} - a,$$

$$z(\varphi) = \frac{2a(1+\sin\theta_A)\cos(\varphi-\theta_A)}{1-\cos\varphi}, \tag{5}$$

where $a'$ is the focal length of the parabola. For nonzero $\theta_A$, we truncate the CPC to a height of $(a'+a)\cot\theta_A$. When we take the limit $\theta_A \to 0$ we find the CPC height as defined in [1] diverges and all UCN paths are contained inside the CPC. In this limit, $a' \to a$ and the equation for a vertical CPC becomes

$$z(r) = \frac{1}{4a}(r+a)^2 - a. \tag{6}$$

The normal to the wall at the point $(r,z)$ is

$$\hat{n} = \frac{(-r-a, 2a)}{\sqrt{r^2+2ar+5a^2}}. \tag{7}$$

In this limit, all orbits of the same velocity originating from the input aperture are bound between extrema of the "neutron fountain" caustic, independent of their initial conditions.

An interesting property that has been exploited in neutron optics is that a gravitational parabolic path originating at the focus of a parabolic surface will reflect to a conjugate parabolic path that also intersects the focus. Steyerl [9] called this property the "neutron fountain." As a consequence, points in the neighborhood of the focus of a paraboloid are self-conjugate so the focal plane is imaged back on to itself. Steyerl and Frank used this property to design imaging systems and microscopes [10]. We use this property to show that a compound parabola can efficiently redirect UCN upward as with UCN microscopes but rather than rotate the parabola about its own axis as is done with UCN imaging optics, we place the focus of one parabola coincident with a reflected parabola, forming a compound parabola, and then rotate about the axis of symmetry (see Figs. 1 and 2).

For 2D massive particles, it is not generally true that all imaged paths take equal time [7,8], but for the case of the "neutron fountain", the orbit time from the focus to the opposite parabola wall and reflecting back, is given by

$$\oint dt = \frac{2v_a}{g} \tag{8}$$

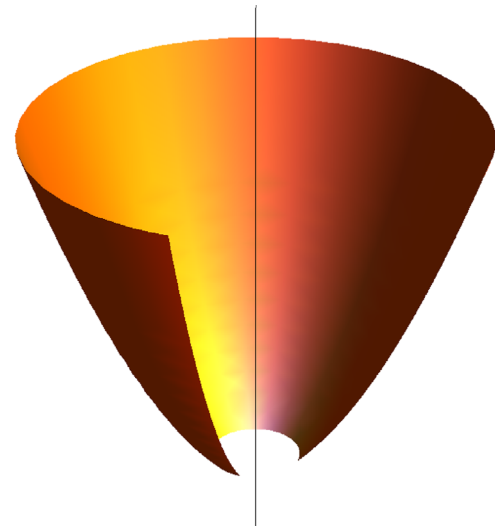

**Fig. 1.** A CPC with a slice cut out.

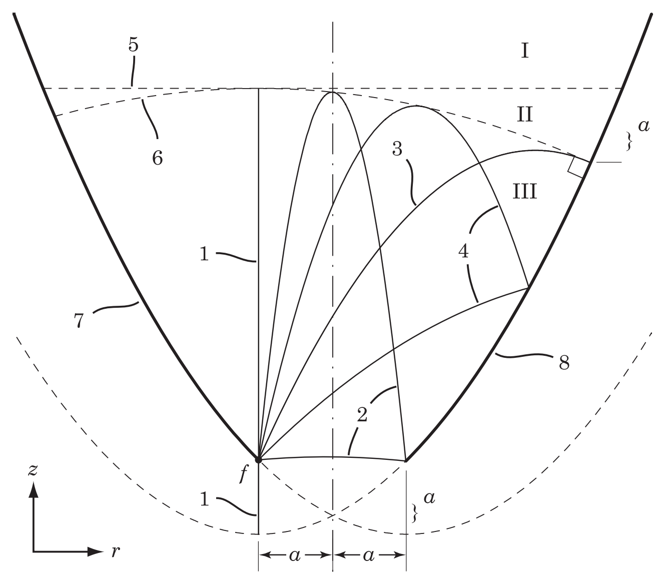


**Fig. 2.** The construction of a CPC using the "neutron fountain" property. For each particle with $v=v_0$, originating from the focus, these edge orbits are stationary to variations from Fermat's principle and to orbital period. Examples shown are (1) a vertical path from the focus, (2) a focus to focus reflection orbit, (3) the self-conjugate orbit, (4) an orbit with initial velocity with $v_r = v_z = v_0/\sqrt{2}$, and (5) the classical limit of an path originating at the aperture at $z=0$. The compound parabola is formed by (7) and the reflection (8). In region I, with $z > v_0^2/g$, UCN are classically forbidden. All UCN from $z=0, r\in[0,a]$ will reach region II. And region III is bounded by the extrema of the edge orbits below 6, the ballistic umbrella of the point $f$.

where $v_a^2 \equiv v_0^2 + 2ag$, where $v_0$ is the initial velocity. This can be compared to a vertical orbit which has orbit time $2v_0^2/g$. This time depends only on the parabola's focal length $a$ and the UCN initial velocity magnitude $v_0$, and is independent of the initial angle to the vertical axis.

For the flight path from the focus $(z=0)$ with an initial velocity $(v_r, v_z) = (v_0 \sin\theta, v_0 \cos\theta)$, the time to reach the parabolic reflector from the focus is

$$\frac{2a}{v_a - v_z}. \tag{9}$$

For each initial vertical velocity, $v_z = v_0 \cos\theta$, for a path starting at focus $z=0$ and angle $\theta$ from the vertical axis, there is a reflected