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Effect of the lattice octupole fields on the synchro-betatron mode coupling instability

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ABSTRACT

Within the framework of a simplifying model and in the first approximation of the perturbation theory we discuss the effect of the Landau damping on the synchro-betatron mode coupling instability of a single bunch in a storage ring. We assume that the required by Landau damping frequency spreads of the betatron and of the synchro-betatron modes are provided by the octupole nonlinearity of the ring lattice focusing. We also assume that the wakefields of the bunch decay substantially during the revolution period of particles along the closed orbit. For this reason, the memory of the bunch wakefields are ignored in this paper.

1. Introduction

Starting from the paper [1] it was understood that the number of particles, which can be collected in a single bunch in many storage rings, can be limited by single-bunch and single-turn instabilities of transverse coherent oscillations of the bunch due to the coupling of its synchro-betatron modes. If the bunch wakes are described by a very wideband transverse impedance $Z^{\perp}(\omega)$, then within the simplest approach, when only two neighbor betatron and synchro-betatron modes of a monochromatic bunch are coupled, coherent oscillations are stable within the band (e.g. in Refs. [1] or [2])

$$|w| = \left|\frac{\Omega_{m_y}}{\omega_s}\right| < \frac{1}{1+8/\pi^2} \simeq 0.552. \quad (1)$$

Here, Ω_{m_y} is the coherent frequency shift of vertical betatron coherent oscillations of the bunch with zero length, ω_s is the frequency of synchrotron oscillations. More general study involving interactions of all synchro-betatron modes of a monochromatic bunch (e.g. in Ref. [3]) results in a bit narrower stability band

$$|w| \le w_{th} \simeq 0.54. \quad (2)$$

Numerous attempts to suppress these instabilities using artificial damping systems suffer from the fact that, although these instabilities occur due to single-turn wakefields, they indeed should be classified as dynamic type phenomena with a sum-type of the coupling between the modes of coherent oscillations (e.g. in Ref. [4]). It means that the instability can be suppressed only in the case, when a damping mechanism results in decays of all, or of the most significant, coupled modes.

In these conditions, a natural mechanism to suppress these instabilities can provide Landau damping due to octupole fields of the ring lattice. The frequency spread of betatron oscillations of the bunch particles due to these octupole fields simultaneously provides Landau damping of the coupled both betatron and of synchro-betatron modes of coherent oscillations. A simplified analysis of such a possibility was reported in Ref. [2], where this mechanism was found as a promising one in the region of parameters where the frequency spread of incoherent betatron oscillations $\delta\omega$ exceeds the frequency of synchrotron oscillations ω_s.

In the paper [2] we simplified calculations assuming that the frequencies of e.g. vertical betatron oscillation of particles linearly depend on the action variable of a particle for these oscillations I_y and does not depend on the action variable of the horizontal betatron oscillations of this particle I_x. For Maxwellian octupole fields this assumption holds only in the case, when $I_x = 0$. More generally, the frequency of e.g. vertical betatron oscillations depends on both variables $\omega_y = \omega_y(I_y, I_x)$. These dependences produce the so-called partial frequency spreads of the incoherent betatron oscillations in the bunch. For example, following the papers [5] we shall call the frequency spread $\omega_y(I_y, I_x = 0)$ as the own frequency spread for the vertical betatron oscillations and we shall call the frequency spread $\omega_y(I_y = 0, I_x)$ as the external frequency spread for the vertical betatron oscillations. Variations in the partial frequency spreads, generally, change the distribution functions in the frequencies of the vertical betatron oscillations and, hence, can change the stability conditions of coherent

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oscillations of the bunch as well as the threshold number of particles in the bunch.

In this paper we study such variations for the case, when Landau damping of the coupled betatron and synchro-betatron collective modes occurs due to general octupole fields of the ring lattice.

2. Mode-coupling equations

We simplify calculations assuming that incoherent oscillations of the bunch particles can be described in the smoothed focusing approximation. For the same reason, we also assume that the dispersion function of the ring is equal to zero at the places, where we shall calculate the coupling impedance and/or the strengths of the octupole lenses of the ring lattice. Then, the canonical transformation to the action-phase variables of unperturbed oscillations of the bunch particles is generated using the following formulae (e.g. in Ref. [4]):

$$y = a_y \cos\phi_y, \quad p_y = -\omega_0\nu_{y0}a_y \sin\phi_y$$
$$x = a_x \cos\phi_x, \quad p_x = -\omega_0\nu_{x0}a_x \sin\phi_x, \quad \frac{d\psi_{y,x}}{dt} = \omega_0\nu_{y,x}$$
$$\theta = \omega_0 t + \phi, \quad \Delta p = p - p_0$$
$$\frac{d\phi}{dt} = \omega_0\eta\frac{\Delta p}{p_0}, \quad \phi = \varphi\cos\psi_s, \quad \frac{d\psi_s}{dt} = \omega_0\nu_s$$
$$\phi_{y,x} = \psi_{y,x} + \phi\frac{d\omega_{y,x}}{d\omega_0} = \psi_{y,x} + \phi\left(\nu_{y0,x0} + \frac{\xi_{y,x}}{\eta}\right), \quad \xi_{y,x} = \frac{d\nu_{y,x}}{d\ln p}$$
$$I_y = \frac{p_0\nu_{y0}a_y^2}{2R_0}, \quad I_x = \frac{p_0\nu_{x0}a_x^2}{2R_0}, \quad \eta = \frac{1}{\gamma^2} - \alpha. \tag{3}$$

Here, the symbols y and x mark the values related to the vertical (y) and to the horizontal betatron oscillations of particles, $\Pi = 2\pi R_0$ is the perimeter of the closed orbit, the suffix 0 marks the values, calculated for the synchronous particle, $E_0 = \gamma Mc^2$ is the particle energy, $\nu_{y,x,s}$ are respectively the tunes of the betatron and of the synchrotron oscillations of particles and α is the momentum compaction factor of the ring.

Due to nonlinear dependencies of the lattice focusing, or defocusing forces on the particle offsets from its position on the closed orbit, the frequencies $\omega_{y,x}$ and ω_s may depend on the amplitudes of the particle betatron and synchrotron oscillations. In this paper we simplify the calculations assuming that the bunch length is short enough to enable neglecting the nonlinearity of the incoherent synchrotron oscillation. It means that in our calculations we take that the frequency ω_s has the same value for all particles of the bunch. Concerning the frequencies of the betatron oscillations of the particles we assume that those linearly depend on the squares of the oscillation amplitudes $2R_0I_{y,x}/(p\nu_{y0,x0})$. For example, for the vertical incoherent betatron oscillations we shall write

$$\omega_y = \omega_0\nu_{y0} + aI_y - bI_x \tag{4}$$

where the values a and b are determined by the strength of the lattice octupole field.

We consider the case of the vertical dipole betatron and synchro-betatron coherent oscillations. For simplicity we also assume that the interaction of the bunch with its surroundings can be described in terms of a wide-band localized transverse coupling impedance and that the bunch wakefields completely decay during a single turn. As usually, we take that the bunch without coherent oscillations is described using the distribution function

$$f = \frac{f_0(I_y, I_x)\rho(\varphi)}{(2\pi)^3}. \tag{5}$$

We assume that the functions $f_0(I_y, I_x)$ and $\rho(\varphi)$ obey the following normalizations conditions:

$$\int_0^\infty dI_x \int_0^\infty dI_y\, f_0(I_y, I_x) = 1, \qquad \int_0^\infty d\varphi\, \varphi\rho(\varphi) = 1.$$

The vertical dipole coherent oscillations of the bunch are described by a small addition of δf to f. Assuming a study of the stability condition problem, we write

$$f = \frac{f_0(I_y, I_x)\rho(\varphi)}{(2\pi)^3} + \sqrt{I_y}\frac{df_0}{dI_y}\sum_{m=-\infty}^{\infty}\chi_m(\varphi)\exp(im_y\psi_y + im\psi_s - i\omega t) \tag{6}$$

where $m_y = \pm 1$. If the perturbations of particle oscillations by the bunch wakes result in reasonable weak variations of amplitudes of coherent oscillations during the periods of incoherent betatron oscillations of particles, the linearized Vlasov equation for δf enables to find out that the amplitudes χ_m obey the following system of integral equations (see, e.g. in Ref. [4]):

$$\chi_m(\varphi) = \rho(\varphi)\int_{-\infty}^{\infty} dn\, \Omega_m(n) J_m(\varphi[n+\zeta]) \times \sum_{m'=-\infty}^{\infty} F(\Delta\omega_m - m'\omega_s)\int_0^\infty d\varphi'\, \varphi' J_{m'}(\varphi'[n+\zeta])\chi_{m'}(\varphi'). \tag{7}$$

Here, $\zeta = m_y\xi_y/\eta$,

$$F(\omega) = -\int_0^\infty dI_x \int_0^\infty dI_y \frac{I_y(\partial f_0/\partial I_y)}{\Delta\omega_m - m_y(aI_y - bI_x)}, \quad \mathrm{Im}\,\omega > 0 \tag{8}$$

$\Delta\omega_m = \omega - m_y\omega_0\nu_{y0}$, $\Omega_m(\omega)$ is the value of the coherent frequency shift of a coasting beam which has the same number of particles as our bunch and which interacts with the same coupling impedance

$$\Omega_m(\omega) = im_y\frac{Ne^2\omega_0}{4\pi p\nu_{y0}}Z_\perp(\omega) \tag{9}$$

and $J_m(x)$ is the Bessel function. In Eq. (7) we used an assumption that the bandwidth of the impedance substantially exceeds the revolution frequency of the bunch particles. For this reason, an exact value of the frequency in the argument of $\Omega_{m_y}(\omega)$ in Eq. (7) was replaced by a combination frequency from the unperturbed spectrum of coherent oscillations $\omega = \omega_0(n + m_y\nu_{y0})$, while the summation over discrete harmonics of the revolution frequency $n = \omega/\omega_0$ was replaced by the integration over continuous harmonic numbers (e.g. in Ref.[4]).

Eq. (7) describes coherent oscillations of a bunch with coupled betatron and synchro-betatron modes. It is clear that such a coupling will be strong only in cases, when the coherent frequency shift of the bunch $\Delta\omega_m$ becomes comparable or higher than the frequency of synchrotron oscillations of particles ω_s. Typically, these general integral equations are too complicated to enable their solution in an analytic, or in a numerical form. In order to avoid this embarrassment and to focus on the effect of Landau damping on the stability of the coupled synchro-betatron modes of the bunch, we consider a simplified model [2], where all particles of the bunch have equal amplitudes of the synchrotron oscillations

$$\rho(\varphi) = \delta(\varphi^2 - \varphi_0^2) \tag{10}$$

and where

$$\Omega_m(n) = \frac{i\Omega_{m_y}}{\pi(n + i\Delta)} \tag{11}$$

while the ring chromaticity can be taken as zero ($\xi_y = 0$). With these assumptions and as far as

$$\chi_m(\varphi) = C_m\delta(\varphi_0^2 - \varphi^2) \tag{12}$$