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Velocity distribution in Recoil-Distance Doppler-Shift experiments

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ABSTRACT

The Recoil-Distance Doppler-Shift (RDDS) or Plunger technique is a well established method to measure lifetimes of excited nuclear states in the pico-second range. In standard RDDS experiments, the velocities of the nuclei of interest emerging from a usually thin target foil are distributed around a mean velocity $\overline{v} = \langle v \rangle_v$ with a relatively narrow width and it is sufficient to assume that all nuclei move with the average velocity. In this paper we investigate the influence of a broader velocity distribution especially for lifetimes τ determined using the DDCM and its basic relation $\tau = -(R(x) - R_{feed}(x))/(\mathrm{d}R(x)/\mathrm{d}x)v$ and simulated experimental data ($R(x)$ decay curve of the level of interest, $R_{feed}(x)$ feeding decay curves). It turned out that it is favorable to use $\langle 1/v \rangle_v$ instead of $1/\langle v \rangle_v$. Further, deviations from the correct lifetimes practically vanish at target to stopper separations close to the maximum amplitude of the function $\mathrm{d}R(x)/\mathrm{d}x$. As a consequence in a plunger experiment target-to-stopper separations should be selected symmetrically around the maximum amplitude of the function $\mathrm{d}R(x)/\mathrm{d}x$ in order to minimize the effect of a broad velocity distribution.

1. Introduction

The Recoil-Distance Doppler-Shift (RDDS) technique is a well-established method to measure lifetimes of excited nuclear states in the pico-second region [1]. In standard RDDS experiments, the velocities of the nuclei of interest after emerging from the target foil are usually distributed around a mean velocity $\overline{v}$, depending on the reaction mechanism, the target thickness, and the velocity distribution of the beam before the target. In a standard analysis of these experiments, the mean velocity is used to obtain the lifetimes of the excited states of interest. The influence of this velocity distribution on the extracted lifetimes and its consideration in the analysis of such experiments are investigated in this work. It will be shown that the determined lifetime depends on the velocity distribution and the measured target-to-stopper distances. The influence of the velocity distribution is discussed for the analysis of a singles γ case by means of the Differential Decay Curve Method (DDCM) [2], but the considerations made here are generally inherent to all RDDS experiments as well as other experiments employing the Doppler-shift, e.g. the Doppler-shift attenuation method.

Previous works (see e.g. Refs. [1,3]) investigated the change of the line-shape of the γ energy spectrum and the resulting change of the peak areas that are used to determine the lifetime. Also the change in the peak area and shape of the shifted component due to a velocity distribution was investigated in these works as well as in Ref. [4]. But the principal influence of the velocity distribution on the lifetime, which occurs by transferring the observables measured at different distances x onto time dependent variables, has so far not been investigated and has been neglected in many RDDS analyses. In this paper the underlying physical functions as well as the quantitative influence of the velocity distribution are investigated. In particular, it will be shown that for typical RDDS experiments the influence is minimized under two conditions: first, the distances are distributed symmetrically around the maximum amplitude of the derivative of the decay function $R(x)$; second, the mean inverse velocity $\langle 1/v \rangle_v^{-1}$ is used instead of the mean average velocity $\langle v \rangle_v$.

2. Observables in RDDS experiments

In standard RDDS experiments, the state of interest is populated by a reaction in a target. The excited nucleus (recoil) leaves the target with a velocity v and is stopped after a certain distance x downstream of the target in a stopper foil. The γ rays are observed with a Doppler-shifted energy for decays that take place in-flight and with an unshifted γ energy for decays in the stopper. The lifetime can then be extracted by the ratio of the intensities of the shifted and unshifted peaks, if the velocity v of the particle and the distance x between the target and stopper foil are known. If the intensities are measured at several distances x, the lifetime may then be derived using the DDCM. In the case of a 'singles' γ

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experiment, the lifetime for a level i is then given by

$$\tau_i(x,v) = \frac{R_i(x,v) - \sum_k b_{ki} R_k(x,v)}{v \frac{d}{dx} R_i(x,v)} \tag{1}$$

where $R_i(x,v)$ is the intensity of the stopped component of the level of interest, $R_k(x,v)$ the intensities of the feeding transition, b_{ki} the branching ration of the transition from the respective feeder k to level i, and v is the velocity after the target [1]; the same notation as in Ref. [1] is used here.

For simplicity, the feeding is assumed to be zero in the discussion here, i.e. $R_i(x,v) = R(x,v)$, $R_k(x,v) = 0$; for the calculation of the examples, which are presented further below, the feeding is correctly included in the calculation.

The term $R(x,v)$ and its derivative depend on the velocity, as the underlying physical functions, i.e. the decay functions, depend on the time. The intensities as a function of t are not directly experimentally accessible. Rather, in an RDDS experiment the following quantity is measured:

$$\frac{\int dv\, f(v) R(x,v)}{\int dv\, f(v)} = \langle R(x,v) \rangle_v. \tag{2}$$

The total measured intensity of one component comprises the statistics of all measured events, each one with its individual velocity v. However, using the term in Eq. (1) assumes that the velocity distribution $f(v)\, dv$ cancels out:

$$\frac{\int \tau(x,v) f(v)\, dv}{\int dv\, f(v)} = \langle \tau(x,v) \rangle_v \stackrel{?}{=} \tau(x) \tag{3}$$

Before investigating this further, one point shall be elaborated in a bit more detail for clarification. The velocity distribution $f(v,x) = f(v)$ is the same for all target-stopper distances x. Nevertheless, the shifted component, in particular its width $\sigma_{Shifted} = \sigma_{Shifted}(x)$, depends on x. Hence, the *observed* velocity distribution in terms of the width of the shifted peak depends on x. This x dependence results from the transformation $R(t) = R(t = x/v)$, where different values are obtained for one particular distance x, if the velocity is not constant. Since v is distributed around $\overline{v}$, it follows that a distribution $\tilde{f}_x(t)$ in time is obtained around the mean value $\overline{t} = x/\overline{v}$. This distribution depends linearly on x and is the origin of the observed dependence of the shifted component on the target-stopper distance. However, it is emphasised that $\tilde{f}_x(t)$ causes the observed x-dependence of $\sigma_{Shifted}(x)$, whilst $f(v)$ remains unchanged for every distance x. The velocity distribution is determined mainly by the reaction mechanisms in the target and the target thickness. The apparent change of v as observed in $\sigma_{Shifted}(x)$ with the distance as discussed e.g. in Ref. [4] is inherently included in the discussion presented in this paper.

In a DDCM analysis, the lifetime τ is calculated with the measured quantities as in Eq. (2). An interesting possibility to avoid influence from the velocity distribution is given for $\gamma\gamma$ coincident cases with high statistics. For these cases, it is possible to select precise velocities from the range of the velocity distribution by applying a narrow energy gate on a part of the shifted peak and thereby suppressing the influence of the velocity distribution (cf. Ref. [5]). Furthermore, the possibility to employ Monte-Carlo simulations, which use an individual velocity for each event, shall be mentioned here (cf. Ref. [3]). This technique also allows to avoid the obstacles discussed in this paper.

However, in a 'standard' RDDS analysis with moderate statistics, the total peak area of the unshifted and shifted component is used in the analysis, as smaller energy gates would inhibit any decent analysis. This is equivalent to taking the weighted mean with respect to the velocity:

$$\tilde{\tau}_{exp}(x) = -\frac{\langle R(x) \rangle_v}{\langle v \rangle_v \cdot \frac{d}{dx} \langle R(x) \rangle_v}. \tag{4}$$

However, this term is not exact. The correct value for τ would be

$$\tau(x) = \left\langle -\frac{R(x)}{v \frac{d}{dx} R(x)} \right\rangle_v. \tag{5}$$

The question arising is whether this inequality has a measurable effect on the resulting lifetime. In the following section, this will be discussed in detail.

3. Typical examples

In the previous section it was shown that the velocity distribution of the recoils after the target affects the outcome of a DDCM analysis. This shall be investigated and presented here by several explicit examples. Three different level schemes L0, L1, and L2 are calculated, as well as three different velocity distributions. The velocity distributions are Gaussian distributions with a mean velocity of $\overline{v} = 20$ and different widths $\sigma = \overline{v}/20, \overline{v}/4, 3\overline{v}/4$. No units are used in this paper for simplicity. Typical dimensions are micrometer and picoseconds.

The level scheme is illustrated in Fig. 1 and the lifetimes for the different level schemes are listed in Table 1. To omit the problem of infinities in the solutions of the Bateman equations for the level scheme L2, the lifetimes were varied slightly (by $\delta\tau = 0.0001$) for the calculation. For the calculation, the velocity distribution is discretised and seven velocities are used for the calculation: $\{\overline{v}, \overline{v} \pm \sigma, \overline{v} \pm 0.3\sigma, \overline{v} \pm 0.6\sigma\}$, which was found to be sufficient. Each velocity is weighted according to the Gaussian distribution. In all cases the initial population is 100% in the highest level 4. The state of interest, i.e. whose lifetime will be computed, is level 1.

Using the parameters given above, the decay functions $R(x)$ and its derivatives are calculated for each level scheme and each discrete velocity (cf. Appendix A for details). For level scheme L0, these functions are illustrated in Fig. 2 for the different velocity distributions. The functions are shown for three velocities $\{\overline{v}; \overline{v} \pm \sigma\}$, as indicated by the blue dots on the velocity distributions, which are illustrated in the smaller insets of the figure. Further, the experimentally observable function, i.e. the average

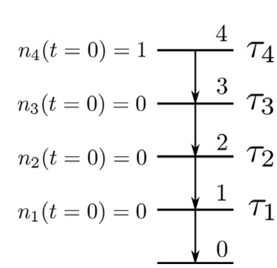

Fig. 1. Level scheme of the calculated examples. The case was calculated for three different sets of lifetimes, which are shown in Table 1.

Table 1
Lifetimes for the calculated examples. No units are used in this paper for simplicity.

Level scheme	τ_1	τ_2	τ_3	τ_4
L0	3	2	1	0.5
L1	2	3	4	5
L2	1	1	1	0.5