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A Kalman Filter approach for track reconstruction in a neutrino telescope

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ABSTRACT

In high energy neutrino telescopes, the detection principle relies on the detection of Cherenkov light emitted from an up-going muon induced by ν_μ that have penetrated the Earth. In the muon energy range of interest for astrophysical searches (namely from about 100 GeV to about 1 PeV), the electromagnetic showers accompanying the muon track generate Cherenkov light emitted within a few degrees of the cone associated to the primary particle. Furthermore, because of photon scattering in the water, the measurement is affected by non-Gaussian noise. Consequently, the track reconstruction in underwater Cherenkov neutrino telescopes is strongly complicated. Moreover, environmental background originates large noise counting rate. In an undersea neutrino detector, in fact, the decay of radioactive elements, mainly the β-decay of potassium isotope ^{40}K, generates electrons that produce Cherenkov light leading an isotropic background of photons. Therefore, the hit-pattern identification of neutrino induced event is non-trivial and the track reconstruction has to deal with a non-linear problem due to this non-Gaussian measurement noise. In this paper a method, based on the Gaussian Sum Filter algorithm to take into account non-Gaussian process noise, for track reconstruction in a km^3 underwater neutrino telescope, is presented.

1. Introduction

In neutrino telescopes track reconstruction is strongly complicated due to the electromagnetic showers accompanying high energy muons [1,2] that generate Cherenkov light. Furthermore, the measurement is affected by non-Gaussian noise related to the photon scattering in the water. Moreover, in an undersea neutrino detector, the decay of radioactive elements in the water, mainly the β-decay of potassium isotope ^{40}K, generates electrons that produce Cherenkov light with a large background counting rate.

As a result, we have a non-linear problem with a non-Gaussian measurement noise. A method for track reconstruction based on the Kalman Filter [3,4] approach is presented as well as a Gaussian Sum Filter algorithm [5] that takes into account non-Gaussian process noise.

2. The Kalman Filter for track fitting

The Kalman Filter is an efficient algorithm for track filtering and fitting specially adapted to sequential measurements [3,4,6]. It works in a two-step process: in the *prediction step*, it produces estimates of the true unknown values, along with their uncertainties. Once the outcome of the next measurement is observed, these estimates are updated using a weighted average (*measurement step*).

The track parameters evaluation proceeds progressively including information of each additional measurement, thus improving iteratively the knowledge on the current track parameters. The track is regarded as a dynamic system described by the state vector $\overline{x}$ uniquely describing the track in each point of its trajectory.

In the Kalman Filter approach, the evolution of the state vector is described by a *system equation*, where the state vector is extrapolated from the hit $k-1$ to the hit k by means of the track model

$$\overline{x}(z_k) = \overline{x}_k = f_k(\overline{x}_{k-1}) + w_k \tag{1}$$

where $\overline{x}_k$, denoting the system state vector *at the hit k*, i.e. after inclusion of k measurements, contains the parameters of the fitted track; f_k is the track propagator from hit $k-1$ to hit k; w_{k-1} incorporates a random disturbance of the track due to multiple scattering or energy loss.

The measured state vector is obtained from the quantities measured by the k-th hit, m_k, that are functions of the state vector, corrupted by a measurement noise ε_k. This is described by the *measurement equation*:

$$m_k = h_k(\overline{x}_k) + \varepsilon_k. \tag{2}$$

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Within the Extended Kalman Filter (EKF) approach, we can write

$$\overline{x}_k = F_k \overline{x}_{k-1} + w_k, \tag{3}$$

$$m_k = H_k \overline{x}_k + \varepsilon_k. \tag{4}$$

The evaluation of the prediction is performed by using the *prediction equation*

$$\overline{x}_{k|k-1} = F_k \overline{x}_{k-1}$$
$$C_{k|k-1} = F_k C_{k-1} F_k^T + Q_k \tag{5}$$

where the covariance matrix $C_{k|k-1}$ at the hit k is obtained by extrapolating the covariance matrix evaluated by using the previous $k-1$ measurements (*predicted error matrix*). Q_k is the covariance matrix relative to w_k and denotes the additional error introduced by the *process noise*: the covariance matrix of the process noise between hit k and hit $k+1$ is added to the propagated covariance matrix. Because of its recursive nature, the filter can run in real time using only the present input measurements and the previously calculated state. The system state vector at the last filtered point contains the full information from all points.

3. Muon tracks in a neutrino telescope

In a neutrino telescope, the neutrinos are detected indirectly by the Cherenkov light emitted from secondary leptons produced in a charged current neutrino interaction ($\nu_l + N \to l + X$) or from the shower generated in a neutral current interaction ($\nu_l + N \to \nu_l + X$). The detection is based on the measurement of the intensity and of the arrival time of Cherenkov light produced along the muon track on a three-dimensional array of Photo-Multiplier Tubes (PMTs). In the energy range of interest for neutrino telescopes, high energy muons can produce electromagnetic radiation through bremsstrahlung. In addition, the arrival time of the photons on the PMTs is smeared by the light-scattering that strongly depends on the distance of the PMT from the track. Therefore, the arrival time depends on the track parameters (time, direction and position) as well as on the PMTs position (i.e. the detector geometry) and track reconstruction deals with a non-linear problem. On top of that, uncorrelated signals, like optical background from ^{40}K decay or genuine electronic noise also affect track reconstruction. These noise signals can be efficiently rejected by a Kalman Filter technique. Indeed, it allows one to use locally the full information in order to associate to each hit the probability to belong to a track.

4. The KM3NeT neutrino telescope

The European KM3Net consortium [7] is performing extensive studies towards the design and realization of a km^3 neutrino telescope in the Mediterranean Sea. The ANTARES [8], NEMO [9] and NESTOR [10] Collaborations, which have developed pilot projects, are committed in the optimization of the Mediterranean neutrino telescope geometry.

4.1. The reference detector

The detector configuration [11] we studied is the one developed by the NEMO Collaboration. It consists of a square array of structures, called towers, each made of a sequence of "storeys" hosting the PMTs. Each storey is rotated by 90°, with respect to the upper and lower adjacent ones, around the vertical axis of the tower. The detector is a square array of 9×9 towers spaced 140 m, with 72 PMTs for each tower, namely 5832 PMTs for the whole detector volume of ~ 0.9 km^3. Each tower is composed of 18 storey, each made of a 20 m long beam structure hosting two optical modules (OM), one downlooking and one looking horizontally, at each end (four OMs per storey). The vertical distance between storeys is 40 m. A spacing of 150 m is added at the base of the tower, between the anchor that fix the structure to the seabed and the lowermost storey. The methods presented in the following can obviously be applied to any detector configuration.

4.2. Detector and track simulation

A dedicated Monte Carlo simulation of the detector and of the track propagation, developed within the ANTARES and NEMO Collaboration [12,13], has been used to assess the performances of the track reconstruction algorithm. The influence of absorption and scattering of light is taken into account by a model that has been tuned onto the data acquired during the measurements at the Capo Passero site [14], in the Ioian Sea, proposed by the NEMO Collaboration for the deployment of a km^3 telescope. The background photons have a measured rate of 30 kHz mainly due to the β-decay of ^{40}K.

The simulated muon flux is distributed as E^{-1} within the range from 100 GeV to 10^4 TeV. Muons are going upward and distributed uniformly in the hemisphere. The starting points of the tracks are always placed outside the detector.

5. Kalman Filter approach for track reconstruction in a neutrino telescope

5.1. The track model

In the Kalman Filter approach, the evolution of the state vector depends on track parameters via the track model f_k. In the case under study, we set

$$f_k(\overline{x}_{k-1}) = \overline{x}_{k-1} + w_k \tag{6}$$

since, for energetic muons, multiple scattering has a negligible effect, the track is considered as a straight line; moreover, the energy loss is not taken into account at this step, thus the process noise w_k is neglected. The predicted error matrix $C_{k|k-1}$ is equal to the filtered error matrix C_{k-1} for $\overline{x}_{k-1}$, since the matrix of derivatives F_k in Eq. (3) is a unit matrix in our case. Conversely, the measurement m_k, corresponding to the hit time, depends on the track parameters $\overline{x}_k$ via Eq. (2), in which the measurement function $h_k(\overline{x}_k)$ can be parameterized as

$$h_k(\overline{x}_k) = T_{theor} \tag{7}$$

T_{theor} is an estimation of the hit time obtained from pure and simple geometrical values for given track parameters and hit position based on the relationship

$$(t_j - t_0) = \frac{1}{c}\left(l_j - \frac{d_j}{\tan\theta_C}\right) + \frac{1}{v_{ph}}\frac{d_j}{\sin\theta_C} \tag{8}$$

where d_j is the distance of the closest approach from the PMT j to the muon trajectory and l_j is the distance travelled by the track. v_{ph} indicates the group velocity of light in water.

In this work, the measurement errors ε_k distribution are based on a detailed simulation of the track and photon propagation and signal detection [12,13].

5.2. Kalman filtering with shifted mean (KFS)

Since the Extended Kalman Filter strategy requires linear approximation of the track model only over a short range, it is suitable for measurements in a neutrino telescope where the