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Effect of large momentum spread on emittance measurements

M. Olvegård *, V. Ziemann

Uppsala University, P.O. Box 516, SE-751 20, Sweden

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ABSTRACT

We discuss the systematic errors in emittance measurements with quadrupole scans and four screens due to large momentum spread in the beam. This is particularly relevant in the drive beam complex of CLIC and the test beam line TBL in the CTF3 facility at CERN. We also discuss methods to adapt the model to correct for the systematic errors.

1. Introduction

The proposed future compact linear collider CLIC [1] will be based on a two-beam acceleration scheme, where a high intensity beam with the moderate energy of 2.4 GeV will be used to generate microwaves that are subsequently used to accelerate a second beam to energies in excess of 1 TeV. The beam that provides the energy is commonly called the drive beam and it is decelerated by up to 90% in so-called power extraction and transfer structures (PETS). This deceleration process induces a large energy spread in the drive beam and also increases the emittance. It is therefore vital to consider diagnostic methods that permit the accurate characterization of these quantities. The increase in energy spread was already addressed in Ref. [2] and in this report we address emittance measurements and how it is affected by the large momentum spread. In quadrupole scans in particular the strength of the quadrupole is varied and thereby also the chromatic effects are different at different quadrupole settings, leading to an effect of the momentum spread on the measured spot size. If uncorrected, this will lead to a misinterpretation of the emittance. In Ref. [3] this effect was addressed in a perturbative way, which is adequate in the case of a Gaussian momentum distribution with an rms spread of a few percent, but in the CLIC drive beam decelerator and even in the currently operated test beam line (TBL) in the CLIC test facility CTF3 at CERN, the momentum spread can easily exceed tens of percent and the profile is vastly different from Gaussian, which warrants a careful analysis, of both the non-perturbative regime as well as non-Gaussian beams. However, chromatic effects play a role not only in quadrupole scans but also emittance measurements where three or four screens placed in a FODO beam line are susceptible to the same misinterpretation and we will consider that setup as well.

In the remainder of this report, we therefore first introduce the emittance measurement setup with two adjacent quadrupoles that permit to have a beam waist on a beam screen and vary the beam size around it both in the horizontal and the vertical plane. In the analysis, we will introduce the energy as an extra parameter and will find the chromatic effects easily parameterized by a small number of integrals over the momentum distribution of the beam. We then analyze the system by assuming a Gaussian momentum distribution and then consider a typical energy distribution in TBL at CERN. We progress to show how the chromatic effects can be compensated provided that the momentum spread is known. We then perform the same type of analysis for an emittance measurement setup where four screens are placed in a FODO beam line and conclude the report with a discussion.

2. Two-quadrupole model

We consider a thin-lens model of the quadrupole scan emittance measurement setup. This is appropriate as long as the lengths of the quadrupoles is small compared to the beta functions, which normally is the case. Despite the high beam intensity, we neglect the defocusing effect of space charge because the emittance is large. Using the Kapchinskij–Vladimirskij model as presented in Ref. [4], we have estimated it to be two orders of magnitude weaker than the typical focusing in the quadrupoles and we therefore neglect it. The simplified beam line consists of a quadrupole with nominal focal length f_1, a drift space of length

* Corresponding author. Tel.: +46 184713253; fax: +46 18 471 5999.
E-mail address: maja.olvegard@physics.uu.se (M. Olvegård).

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l_1, another quadrupole with focal length f_2 followed by a drift of length l_2 and the beam screen. The emittance measurement is based on the determination of the beam matrix σ immediately upstream of the first quadrupole by observing the beam size on the screen while varying the quadrupoles. We assume that we know the initial sigma matrix, predict the measured beam size and then invert the in general over-determined linear system in the least-squares sense. For this analysis, we thus need the transfer matrix R from the first quadrupole to the beam screen

$$R = \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{l_2}{f_2}-\frac{l_1}{f_1}-\frac{l_2}{f_1}+\frac{l_1 l_2}{f_1 f_2} & l_1+l_2-\frac{l_1 l_2}{f_2} \\ -\frac{1}{f_1}-\frac{1}{f_2}+\frac{l_1}{f_1 f_2} & 1-\frac{l_1}{f_2} \end{pmatrix}. \quad (1)$$

For computational ease, we have used the same sign for both quadrupoles, but in the final equations it is of course trivial to flip the sign of the defocusing quadrupole. Note also that we consider one plane, horizontal or vertical, at a time. Observe that the elements of the transfer matrix R are functions quadratic in the focal strengths $1/f_i$. Once we know the transfer matrix as a function of the focal strengths, we can calculate the beam size on the screen Σ from

$$\Sigma^2 = R_{11}^2\sigma_{11} + 2R_{11}R_{12}\sigma_{12} + R_{12}^2\sigma_{22} \quad (2)$$

where σ_{ij} are the matrix elements of the sigma matrix upstream of the first quadrupole. We observe that the right hand side of Eq. (2) now contains up to the fourth power of the inverse focal lengths.

Now is the time to introduce the chromatic effects by noting that the focal length is inversely proportional to the quadrupole gradient and proportional to the beam energy, hence, it has the momentum dependence

$$\overline{f}(\delta) = (1+\delta)f \quad (3)$$

where we chose to parameterize the momentum by $\delta = \Delta p/p$, the relative difference to the nominal momentum p and f the focal length corresponding to that same momentum.

In order to find the momentum spread dependence of the measured beam size on the screen Σ, we insert the transfer matrix elements from Eq. (1) into the equation for the beam size Eq. (2), augment each focal length by a factor $1+\delta$ and sort the resulting equation into powers of $1/(1+\delta)$. The algebra is somewhat lengthy though simple. We therefore only show one term as an example

$$R_{12}^2 = l_1^2 + l_2^2 + 2l_1 l_2 - \frac{1}{1+\delta}\left(2\frac{l_1^2 l_2}{f_2} + 2\frac{l_1 l_2^2}{f_2}\right) + \frac{1}{(1+\delta)^2}\left(\frac{l_1^2 l_2^2}{f_2^2}\right) \quad (4)$$

and similar expressions for R_{11}^2 and $R_{11}R_{12}$ that enter in Eq. (2) are given in Appendix A.

If we have a beam with a momentum distribution $\psi(\delta)$, we can calculate the rms beam size on the screen Σ by averaging over the momentum distribution $\psi(\delta)$. A justification of this is given in Appendix B. We observe that the entire momentum dependence of the beam size is encoded in integrals of the type

$$I_n = \int^{\infty} \frac{\psi(\delta)}{(1+\delta)^n}\, d\delta \quad (5)$$

where the lower integral boundary has to be chosen to avoid the pole at $\delta = -1$ which corresponds to a case where particles have lost all momentum and will not propagate to the screen anyway. Note that we also assume without loss of generality that the momentum distribution $\psi(\delta)$ is normalized to unity.

3. Gaussian momentum spread

We start by considering the following normalized Gaussian momentum distribution which is characterized by its width Δ

$$\psi(\delta) = \frac{1}{\sqrt{2\pi}\Delta} e^{-\delta^2/2\Delta^2} \quad (6)$$

As a first step we calculate the integrals for small momentum spread $\Delta \ll 1$ which is trivial by noting that the expansion of $1/(1+\delta)^n$ has the following form [5]:

$$\begin{aligned} \frac{1}{1+\delta} &\approx 1-\delta+\delta^2-\delta^3+\delta^4-\delta^5+\delta^6 \\ \frac{1}{(1+\delta)^2} &\approx 1-2\delta+3\delta^2-4\delta^3+5\delta^4-6\delta^5+7\delta^6 \\ \frac{1}{(1+\delta)^3} &\approx 1-3\delta+6\delta^2-10\delta^3+15\delta^4-21\delta^5+28\delta^6 \\ \frac{1}{(1+\delta)^4} &\approx 1-4\delta+10\delta^2-20\delta^3+35\delta^4-56\delta^5+84\delta^6. \end{aligned} \quad (7)$$

Averaging over the momentum distribution $\psi(\delta)$ requires to evaluate the following integrals:

$$I_n(\Delta) = \frac{1}{\sqrt{2\pi}\Delta}\int_{-\infty}^{\infty} \frac{e^{-\delta^2/2\Delta^2}}{(1+\delta)^n}\, d\delta \quad (8)$$

which, using the expansion discussed above, summarizes to

$$\begin{aligned} I_1(\Delta) &\approx 1+\Delta^2+3\Delta^4+15\Delta^6 \\ I_2(\Delta) &\approx 1+3\Delta^2+15\Delta^4+105\Delta^6 \\ I_3(\Delta) &\approx 1+6\Delta^2+45\Delta^4+420\Delta^6 \\ I_4(\Delta) &\approx 1+10\Delta^2+105\Delta^4+1260\Delta^6 \end{aligned} \quad (9)$$

provided that the momentum spread Δ is small such that we can extend the integral boundaries to minus infinity without having to worry about the pole at $\delta = -1$. The general form is derived in Appendix C.

Using these auxiliary functions, we can calculate how the quadrupole scan data are affected by large momentum spread. As parameters we use the following start values that correspond to typical values in TBL at CERN.

Quantity	Value	Units
Energy	150	MeV
ε_n	200	mm mrad
β	8	m
α	-1	
f_1	0.73,…,1.46	m
f_2	-1.46	m
l_1,l_2	0.5, 1.33	m

In Fig. 1 we show the beam size on the screen squared as a function of the strength of the first quadrupole, which is varied, while the second quadrupole is kept fixed at $f_2 = -1.46$ m. The three curves are calculated with momentum spread Δ to be equal to 0, 0.1, and 0.2, respectively. We observe significant variations of the measured beam size for non-zero momentum spread. These differences are more pronounced for larger quadrupole excitation, which is consistent with the fact that chromatic effects are more significant for stronger quadrupole excitations. We also observe that larger momentum spread Δ increases the measured beam size for a given quadrupole excitation.

4. Determining the emittance

The beam sizes Σ measured as a function of the quadrupole excitation which is shown in Fig. 1 is post-processed to determine