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Application of quasi-optimal weights to searches of anomalies. Statistical criteria for step-like anomalies in cumulative spectra

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A B S T R A C T

It is demonstrated how the method of quasi-optimal weights can be applied to searches of anomalies in experimental data. As an example, a convenient statistical criterion is derived for step-like anomalies in cumulative β-decay spectra in the direct neutrino mass measurement experiments. It is almost as powerful as the locally most powerful one and appreciably excels the conventional χ^2 and Kolmogorov–Smirnov tests. It is also compared with an ad hoc criterion of «pairwise correlations of neighbours»; the latter is seen to be less powerful even if more sensitive to more general anomalies. As a realistic example, the criteria are applied to the Troitsk-ν-mass data.

1. Introduction

Controversies around anomalies in experimental data—potential signals of new physics—are quite common. Given that a one-size-fits-all solution does not exist, the next best thing would be to have a more or less systematic approach to constructing correct statistical criteria for confirming/rejecting particular kinds of anomalies. In this paper we show how a very general method of quasi-optimal weights [1] can be extended to problems of this kind (see below for a theoretical background). A non-trivial specific example we consider is borrowed from real experimental practices and is sufficiently rich in detail to provide a comprehensive illustration of the method.

1.1. Theoretical background

In the practical aspect, the method of quasi-optimal weights represents a shift of emphasis from event selection (yes–no decisions) to event weighting wherein events are summed up with continuously varying weights. In the theoretical aspect, it represents a switch from the traditional set-theoretic paradigm as actualised in Kolmogorov's axiomatics of the theory of probability to the modern functional paradigm where probability measures and other "generalised" functions such as Dirac's δ etc., are defined via their integrals with continuous, smooth etc. weights. The method thus has rather deep mathematical roots. The complex of ideas around weighting was emerging in several different application domains: the 1896 paper by Karl Pearson on the method of moments; the early XX c. discovery of the family of Galerkin methods; the discovery of generalised solutions of variational problems in the 1930s by L.C.Young. An early abstraction of these ideas appeared in Daniell's definition of integral (1918), and this line of thought eventually lead to the so-called generalised functions or distributions (S.L. Sobolev, 1936; L. Schwartz, 1945). The first attempt to make these ideas accessible to a broad range of physicists was undertaken by Schwartz [2]. A systematic advocacy of the functional view was presented in [3].

The two paradigms—the set-theoretic (yes–no decisions) and the functional (weighting) ones—only offer different viewpoints on the same network of theorems and formulae. The resulting views, however, are rather different: so a forest that appears naturally chaotic from a road may be seen to contain rows of artificially planted trees once viewed from certain points off the road. An alternative viewpoint can thus help one notice new things not easily seen from a traditional one. For instance, the functional viewpoint made it possible for one of the present authors to discover powerful algorithms for theoretical calculations [4] (for an example of such calculation see [5]). It was then

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natural to address problems in the adjacent field of data processing and to re-examine the conventional methods from the modern functional viewpoint [6,1]; the present paper is part of that effort.

The Troitsk-ν-mass experiment [7,8,9] with its anomaly that remained unexplained for a number of years (for details see below) provided a venue and a stimulus to play with a variety of statistical problems and algorithms. In particular, it was deemed essential to find fully satisfactory procedures, i.e., such that they (1) were correctly expressed in the language of mathematical statistics (rather than be ad hoc formulations often used to express one's intuitions about the problem), and (2) used criteria that are best for the situation (i.e. close to the Rao–Cramer boundary). A concern about proper treatment of non-gaussian background expressed by the late experiment leader V.M. Lobashev prompted a re-examination of the least squares method in Ref. [1] (and eventually led to a new reprocessing of the Troitsk-ν-mass data in Ref. [10]). The present paper addresses another concern, that of a similarly fully satisfactory treatment of the anomaly. Prior to a formal discussion of the anomaly, it is convenient to give some details on the physics context of the Troitsk-ν-mass experiment.

1.2. Physical background

A precise measurement of the endpoint region of the tritium β-spectrum is perhaps the most sensitive way to directly measure the neutrino mass. This is the aim of the Troitsk-ν-mass experiment [7,8,9,10]. On the other hand, the neutrino oscillation experiments (Super-Kamiokande [11], Kamiokande [12] Gallex [13], SAGE [14], SNO [15], Borexino [16], KamLAND [17], K2K [18], T2K[19], MINOS [20], OPERA [21], Double Chooz [22], Daya Bay [23], RENO [24] etc.) are able to measure differences between neutrino masses squared (and mixing angles as well), whereas cosmological observations (see e.g. [25]) and neutrinoless double β-decay measurements (for a review see [26]) are model-dependent types of neutrino mass measurements. However, the direct measurements of the β-decay spectrum are both model-independent and provide estimates of the effective mass of electron antineutrino (for a review of neutrino mass search in tritium β-decay see [27]).

In the Troitsk-ν-mass experiment, the so-called cumulative (integral) spectrum is measured; this means that all electrons with energies no less than a given threshold E are detected and counted (see [10] for a brief description of the experimental set-up).

The first analysis [8,9] of the Troitsk-ν-mass data yielded a rather large and negative value for the neutrino mass squared, $m_\nu^2 \sim -(10/20)\ \text{eV}^2$. This was interpreted as due to an excess of electrons near the end-point energy of the tritium β-decay spectrum; in cumulative spectra, such an excess takes the form of a step. (Although the effect is likely of apparatus origin [8,9,10], for instance due to ill-estimated source thickness, there could be other interpretations, e.g. additional week interactions [28] or tachyonic neutrinos [29].) Such a step is described by two parameters, height and position. Including these into the fit, a satisfactory value for the neutrino mass squared was obtained, $m_\nu^2 = -2.3 \pm 2.5_{fit} \pm 2.0_{syst}\ \text{eV}^2$ [7,8,9].

Recently, a new data analysis has been completed using the method of quasi-optimal weights [1], adapted specifically for the task. As opposed to the least-squares method used in the first analysis, the new method accounts for the Poisson form of the experimental data distribution. Also, the theoretical model of the experimental setup has been improved [10]. As a result of all the improvements, the new analysis yielded a physically relevant value of the neutrino mass squared directly, i.e. without introducing the step.

Although the final publication [10] settled, for reasons of practical expedience, on a procedure that was deemed good enough for its purpose (as if done from point zero, with a minimal heed paid to the anomaly problem encountered in the first analysis), the quasi-optimal criterion we developed remains of interest for a number of reasons.

First, several physical mechanisms were proposed to explain the anomaly (additional week interactions, tachyonic neutrinos etc.), therefore searches for such an anomaly may be an interesting problem on its own right. The χ^2 criterion is not the best tool for that.

Second, since uncertainties of some apparatus parameters proved to contribute significantly to the systematics and affect the anomaly, a criterion of this particular kind may be a useful independent means to control those uncertainties.

Third, the overall situation with anomaly search is rather generic, and even if details (the exact form of the anomaly etc.) may in general vary, the derivation and use of the criterion in this specific case provide a good illustration of the method and may serve as guidance in other cases.

This last point—to provide an illustration of the method—is the focus of this paper.

1.3. Formal description of the anomaly

For definiteness, we consider a simplified version of the problem that occurs in the real Troitsk-ν-mass experiment (where the measured values have Poisson distribution etc.). This will allow us to perform explicit computations while avoiding non-essential complications.

However, the criterion that we are going to construct will be quite general and applicable, e.g. to situations with normally distributed data etc.

Assume that some spectrum $\mu\ (E)$ is to be measured; the control variable E will be called energy for convenience. In the Troitsk-ν-mass experiment it is the retarding potential and μ is the cumulative spectrum that takes into account the geometrical resolution and the energy loss spectrum. The geometrical resolution is a continuous function that replaces the step-like cutoff involved in the definition of the cumulative spectrum; it depends mainly on the configuration of electrostatic and magnetic fields, and it was measured with the electron gun and also derived from a full simulation with realistic field configuration, see Ref. [10] for details.

Suppose the measurements are done for some values E_i, $i=1,..,M$ (M is the number of values E_i in the given set). Suppose for each E_i a number of events for a fixed period of time (the same for all i) is measured and this number is Poisson-distributed with $\mu_i = \mu\ (E_i)$. The anomalous contributions, for which we need a special criterion, have the form of a step (in the Troitsk-ν-mass cumulative spectrum the step is a result of integration of a δ-shaped anomaly):

$$\mu'(E) = \mu(E) + \Delta \cdot \theta(E_{st} - E), \tag{1}$$

where $\mu\ (E)$ is the spectrum without the anomalous contribution, $\theta\ (x)$ is the Heaviside function, E_{st} is the step position and Δ is its height (Δ is the mean of count of the extra electrons with the energy E_{st} over a fixed period of time). In terms of the set E_i, define m according to E_m, $E_m \le E_{st} < E_{m+1}$ (Fig. 1).

We assume that the data are fitted without account of the anomaly. The goal is then to find a powerful, general and handy test which would be sensitive to anomalous contributions of the described type. To compare the criteria, the so-called power functions will be used.