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Effect of two-stream instability on the saturation mechanism of a two-stream free-electron laser with a helical wiggler pump

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ABSTRACT

In this paper we will investigate the effect of two-stream instability on saturation mechanism in Two-Stream Free Electron Laser (TSFEL) with a helical wiggler pump. A two-velocity relativistic electron beam propagates through the helical wiggler field. The relativistic electron stream is assumed to be cold. The slippage of the electromagnetic wave with respect to the electron beam is ignored. Self-consistent evolution of an electromagnetic wave in the presence of two-velocity electron beam is described by a set of coupled nonlinear differential equations in 1D approximation. Slowly varying envelope approximation is used and by the Runge–Kutta method algorithm is solved numerically. The power versus the axial distance has been plotted. It has been found that the TSFEL reaches the saturation regime in a shorter axial distance in comparison with the conventional FEL.

1. Introduction

Using two electron beams rather than one in a free-electron laser has some advantages. For example, when a two-stream instability occurs, it causes an increase in gain as well as the growth rate [1–5]. Two-stream FEL gives the coherence laser radiation and therefore, the problem of longitudinal coherency that occurs in the (Self Amplified Spontaneous Emission) SASE FEL does not occur in this model [6]. Employing of two stream-instability as a basic mechanism in FEL is proposed first by Bekefi and Jakobs [1]. So far, TSFEL with different wiggler pumps with and without axial guide magnetic field have been considered frequently in literatures [7–13]. In addition to an advanced accelerator same as the Multi-Channel Linear Induction Accelerator (MLINAC) or a two-beam accelerator instead of a conventional LINAC, this model of FEL require a merging system (constructed on the basis of magnetic turning systems) [14,16–18]. The merging system formed the two-velocity electron beam. The design of an accelerator's beam source and injector plays a major role in determining a maximum current and brightness. In the MLINAC two electron beams accelerated simultaneously in two different channels. Design elements of the MLINAC and two-beam accelerators are given in [19–21]. Nonlinear theory of two-beam FEL with a tapered wiggler pump and a 3-D simulation of prebunched two-beam FEL with a planar wiggler pump have been studied in Refs. [6,7]. In the present paper we investigate the effect of the two-stream instability on the saturation mechanism of TSFEL with a helical wiggler pump. Energy difference of two beams in TSFEL is an important factor. Generally, in Two-Beam FELs with planar magnetic pump the lower (higher) energy electron beam has a resonant electron energy of $\gamma_1(\gamma_n)$. The nth harmonic wave number satisfied the resonance condition so it can be easily shown that $\gamma_n=\sqrt{n}\gamma_1$ [6,11]. Since we use a helical wiggler pump in our model and there is no possible harmonic generation, so, we choose the optimum difference of the beams energy accordance of the linear growth rate of TSFEL, which, have been obtained in Refs. [22,23]. Wave growth, and thus, electron bunching occurs because of the coupling between the negative energy wave in one stream, and the positive energy wave in the other stream.

A simplified schematic of a typical TSFEL in the amplifier mode configuration is shown in Fig. 1. The set of parameters in Fig. 1 is a common one for numerical analyses, for instance, a helical wiggler of $B_w=2$ kG maximum magnetic field, $\lambda_w=2$ cm length of the wiggler period, $r_b=0.1$ mm beam radius, electron beam with a total current of $I=0.67$ A, $E=0.153$ MeV energy and $\Delta E=0.025$ MeV difference energy of the beams.

The paper is organized as follows: the theoretical model with the electron orbit and the field equations is introduced in Section 2, while the numerical solution of the coupled particle and field differential equations is given in the Section 3. Summary and conclusion are reported in the Section 4.

2. Fundamental equations

2.1. The physical configuration

A two-velocity transversely homogeneous non-neutralized relativistic electron beam with different velocity v_1 and v_2 propagate

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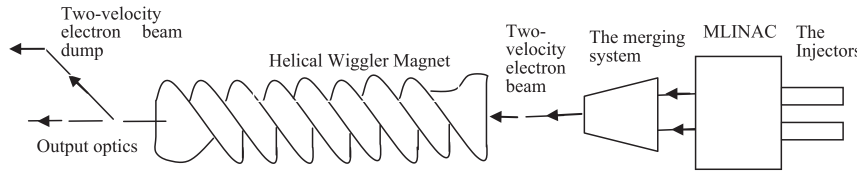

Fig. 1. Schematic diagram of a two-stream free electron laser in amplifier mode, $\lambda_w = 2$ cm, $B_w = 2$ kG.

along the positive z direction trough the magnetic wiggler field, the wiggler field is given by

$$\vec{B}(z) = B_w[\hat{e}_x \cos(k_w z) + \hat{e}_y \sin(k_w z)]. \tag{1}$$

where $k_w (=2\pi/\lambda_w)$ is the wiggler wave number, $\hat{e}_x$, $\hat{e}_y$ are the unit vectors of a Cartesian coordinate system.

The fluctuating electromagnetic and electrostatic fields are treated using the time-varying vector and scalar potentials in the Coulomb gauge, and we assume that these fields are of the form [14]:

$$\delta\vec{A}_i(z,t) = \delta\hat{A}(z)[\hat{e}_x \cos\alpha_{+i}(z,t) - \hat{e}_y \sin\alpha_{+i}(z,t)] \tag{2}$$

$$\delta\emptyset_i(z,t) = \delta\hat{\emptyset}(z)\cos\alpha_i(z,t). \tag{3}$$

here the subscript $i=1,2$ refers to the quantities of the different beams. $\delta\hat{A}_i(z)$ and $\delta\hat{\emptyset}_i(z)$ are the time independent amplitude of the vector and scalar potentials. $\alpha_{+i}(z,t)$ and $\alpha_i(z,t)$ are electromagnetic and space charge phases defined as

$$\alpha_{+i}(z,t) = \int_0^z d\acute{z}\, k_{+i}(\acute{z}) - \omega t \tag{4}$$

and

$$\alpha_i(z,t) = \int_0^z d\acute{z}\, k_i(\acute{z}) - \omega t \tag{5}$$

where ω is the wave frequency, $k_{+i}(\acute{z}) = k_i(\acute{z}) - k_w$ and $k_i(\acute{z})$ are the wavenumbers. This is equivalent to the WKB formulation when it is implicitly assumed that the amplitudes and wavenumbers vary slowly over a wavelength [14].

2.2. The electron orbit equation

The electron orbit equations can be obtained by substitution of the static fields in Lorentz force equation

$$\frac{d\vec{p}_i}{dt} = -e\left[\delta\vec{E} + \frac{\vec{v}_i}{c} \times (\vec{B}_w + \delta\vec{B})\right]. \tag{6}$$

where $\delta\vec{E}$ and $\delta\vec{B}$ are the fluctuating electromagnetic fields which are derivable from the vector and scalar potentials (2 and 3). It is convenient to write equation in rotating frame with the wiggler field as

$$\begin{cases} \hat{e}_1 = \cos k_w z \hat{e}_x + \sin k_w z \hat{e}_y \\ \hat{e}_2 = -\sin k_w z \hat{e}_x + \cos k_w z \hat{e}_y \end{cases} \tag{7-1}$$

$$\hat{e}_3 = \hat{e}_z. \tag{7-2}$$

It has been assumed that the amplitude and phase are slowly varying functions of position ($|\partial\delta\hat{A}/\partial z| \ll |k_+ \delta\hat{A}|$), and this occurs only in the vicinity of the wave particle resonance at $\omega \cong k_+ v_z$. With this assumption Eq. (6) yields three differential equation for three components of momentum like variables $\vec{u}_i = \vec{p}_i/mc$. Using the new dimensionless variables: $\vec{\beta}_i = \vec{v}_i/c$, $\bar{z} = zk_w$, $\bar{t} = tck_w$, $\bar{\varpi} = \omega/ck_w$, $\bar{k} = k/k_w$, these equation can be written as [14]

$$\frac{du_{1i}}{d\bar{z}} = u_{2i} + \frac{d\delta a}{d\bar{z}}\cos\psi_i \tag{8-1}$$

$$\frac{du_{2i}}{d\bar{z}} = -u_{1i} - \frac{d\delta a_i}{d\bar{z}}\sin\psi_i - \hat{\omega}_w \tag{8-2}$$

$$\frac{du_{3i}}{d\bar{z}} = \bar{k}_+ \delta a_i \frac{u_{1i}\sin\psi_i + u_{2i}\cos\psi_i}{u_{3i}} - \frac{1}{\beta_{3i}}\left(\bar{k}\delta\varphi_i \sin\psi_{sci} - \frac{d}{d\bar{z}}\delta\varphi_i \cos\psi_{sci}\right) + \hat{\omega}_w \frac{u_{2i}}{u_{3i}}. \tag{8-3}$$

where $\hat{\omega}_w = eB_w/mk_w c^2$ is the wiggler parameter, and $\delta a_i = e\delta\hat{A}_i(\bar{z})/mc^2$ and $\delta\varphi_i = e\delta\hat{\emptyset}_i(\bar{z})/mc^2$.

Here $\psi(\equiv \alpha_{+i}(\bar{z},\bar{t}) + \bar{z})$ and ($\psi_{sci} \equiv \alpha_i(\bar{z},\bar{t})$ are ponderomotive and space charge phase respectively. They can be written as [15]

$$\frac{d\psi_i(\bar{z})}{d\bar{z}} = \bar{k}_+(\bar{z}) + 1 - \frac{\bar{\varpi}}{\beta_{3i}} \tag{9-1}$$

$$\frac{d\psi_{sci}(\bar{z})}{d\bar{z}} = \bar{k}(\bar{z}) - \frac{\bar{\varpi}}{\beta_{3i}}. \tag{9-2}$$

In the above differential equations we have changed the integration parameter from $\bar{t}$to $\bar{z}$, according to the relation $d/d\bar{t} = \beta_3 d/d\bar{z}$.

2.3. The field equations

In the Coulomb gauge the Maxwell's equations can be written as [14,15]

$$\left(\frac{\partial^2}{\partial\bar{z}^2} - \frac{\partial^2}{\partial\bar{t}^2}\right)\delta a(\bar{z},\bar{t}) = -4\pi\delta J_\perp(\bar{z},\bar{t}) \tag{10-1}$$

$$\frac{\partial^2}{\partial\bar{z}\partial\bar{t}}\delta\varphi(\bar{z},\bar{t}) = 4\pi\delta J_z(\bar{z},\bar{t}). \tag{10-2}$$

where $\delta J(\bar{z},\bar{t})$ is the nonlinear total current density, $\delta J_\perp(\bar{z},\bar{t})$ and ($J_z(\bar{z},\bar{t})$) are the component of the nonlinear current density perpendicular and along the z direction respectively. The current densities can be written as average over the entry time t_0 (defined as the time at which an electron crosses the $\bar{z}=0$ plane)

$$\delta J(\bar{z},\bar{t}) = -\frac{1}{4}\sum_{i=1}^{2}\omega_{bi}^2 \int_{-\infty}^{+\infty} d\bar{t}_0 \beta_i(\bar{t},\bar{t}_0)\frac{\delta[\bar{t} - \tau(\bar{z},\bar{t}_0)]}{|\beta_{zi}(\bar{t},\bar{t}_0)|}. \tag{11}$$

where $\omega_{bi}^2 = 4\pi n_{bi} e^2/mc^2$, $\vec{\beta}(\bar{t},\bar{t}_0)$ is the velocity of an electron at time $\bar{t}$ which crossed the entry plane at time $\bar{t}_0$, and

$$\tau(\bar{z},\bar{t}_0) = \bar{t} + \int_0^z \frac{d\bar{z}}{\beta_z(\bar{z},\bar{t}_0)}. \tag{12}$$

It should be noted that it is implicitly assumed that the electron beam is monoenergetic.

By the substitution of microscopic fields and source current in Eqs. (2 and 3) into Maxwell's Eqs. (10-1 and 10-2) a set of coupled nonlinear differential equations for the slowly varying amplitudes and phases is obtained. The nonlinear Eq. (10-1) can be reduced to three first order differential equations for: δa, Γ_+, and k_+.

$$\frac{d\delta a}{d\bar{z}} \equiv \Gamma_+ \delta a, \tag{13-1}$$

$$\frac{d\Gamma_+}{d\bar{z}} = (-\bar{\varpi}^2 + \bar{k}_+^2 - \Gamma_+^2) + \sum_{i=1}^{2}\frac{\omega_{bi}^2}{\delta a}\frac{u_{1i}\cos\psi_i - u_{2i}\sin\psi_i}{|u_{3i}|} \tag{13-2}$$